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USE OF DREDGINGS FOR LANDFILL

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FOREWORD

The Environmental Protection Agency was created because of increasing public and government concern about the dangers of pollution to the health and welfare of the American people. Noxious air, foul water, and spoiled land are tragic testimony to the deterioration of our natural environment. The complexity of that environment and the interplay between its components require a concentrated and integrated attack on the problem.

Research and development is that necessary first step in problem solution and it involves defining the problem, measuring its impact, and searching for solutions. The Municipal Environmental Research Laboratory develops new and improved technology and systems for the prevention, treatment, and management of wastewater and solid and hazardous waste pollutant discharges from municipal and community sources, for the preservation and treatment of public drinking water supplies, and to minimize the adverse economic, social, health, and aesthetic effects of pollution. This publication is one of the products of that research; a most vital communications link between the researcher and the user community.

The need to protect the ecology of the Great Lakes and the other waterways of the United States has led to a variety of problems concerned with the dredging and disposal of increasing volumes of polluted dredge spoil in areas of high population density and industrial development. One commonly used alternative to open water disposal is to place these polluted sediments in diked containment areas to form landfills of marginal value. However, due to the high costs involved, the scarcity of land, and other environmental and economic considerations, these landfills should desirably serve some useful purpose. Accordingly, this study was directed toward evaluating quantitatively the engineering characteristics of dredged materials as they affect their potential usefulness in a landfill.

Francis T. Mayo, Director
Municipal Environmental Research
Laboratory

MATHEMATICAL MODEL FOR ONE-DIMENSIONAL
DESICCATION AND CONSOLIDATION OF DREDGED MATERIALS

SUMMARY

A mathematical model has been developed to represent the physical phenomena that occur during the desiccation and one-dimensional consolidation of successive layers of dredged materials as they are periodically deposited in a diked containment area. The governing boundary value problem, defined in terms of pore water pressures, consists of two field equations (one for the saturated domain and one for the unsaturated domain), a drainage boundary condition, an evapotranspiration boundary condition, and a series of continuity conditions at the interfaces between different layers. A number of simplifying assumptions were made to render the field equations tractable, and a step-by-step numerical procedure was used to solve the linearized boundary value problem; at the end of each step, the errors introduced by the simplifying assumptions were corrected. Based on a thorough study of the convergence and stability conditions associated with the numerical approximation employed, a system of automatic corrections has been incorporated into the computer program to reduce the time increment if stability problems originate during the solution. The results of an extensive experimental investigation were used to establish empirical relationships for the permeability, water retention, and volume change characteristics of typical dredged materials from the vicinity of Toledo, Ohio, and these relationships were employed in conjunction with the mathematical model (a) to calculate settlements of the materials in the Penn 7 disposal area in Toledo and (b) to evaluate the effect that modifications in the

boundary conditions have on the consolidation behavior of the fill.

Computed consolidation rates were found to agree reasonably well with those measured at the Penn 7 field site, especially during periods of desiccation. Although drainage conditions at the bottom of the layer do exert some influence on the consolidation rate during deposition of the dredgings, the evapotranspiration potential renders this effect almost negligible when desiccation takes place at the surface. Transpiration plays an important role on the consolidation rate during the early stages of desiccation, but this effect is reduced considerably as the water table approaches an equilibrium position. Although significant improvements in the consolidation behavior of dredged materials can apparently not be achieved with low cost modifications in the boundary conditions, the use of vegetation with high transpiration rates and the ability to grow in wet soils appears to offer the most promise as an inexpensive method for accelerating the consolidation rate of dredged materials in a landfill.

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LIST OF SYMBOLS

A	= Reduced time factor
A_j	= Maximum value of a solution at time j
c_v	= Coefficient of consolidation
C	= Rohwer's pan coefficient
C	= Specific water capacity
C_c	= Compression index
C_1	= Coleman's first compressibility coefficient
C_2	= Coleman's second compressibility coefficient
C^*	= Dimensionless specific water capacity
D	= Diffusivity function
e	= Void ratio
e_a	= Air vapor pressure
e_s	= Vapor pressure at the soil surface
e_w	= Vapor pressure at the water surface
e_o	= Initial void ratio
e^*	= Intermediate void ratio during end-of-step corrections
$E_{i,j}$	= Truncation error
E_p	= Evapotranspiration rate
E_{pF}	= Evaporation rate from a free water surface
E_t	= Evaporation rate from a soil surface
$f_R(r)$	= Probability mass function of the rainfall, R
$F_R(r)$	= Cumulative distribution function of the rainfall, R
$F_s(z)$	= Scale heterogeneity function
G_s	= Specific weight density of the solids

h = Dimensionless water head

h_n = Normal derivative of h at a boundary

h_s = Tangential derivative of h at a boundary

h^* = Linearized dimensionless water head

H_f = Thickness of the impeding non-consolidating layer of foundation soil

H_i = Thickness of the i^{th} layer of sediments

i = Space subindex

j = Time subindex

k = Coefficient of permeability

k_c = Coefficient of permeability of the dredged material at the bottom of the deposit

k_f = Coefficient of permeability of the foundation soil

k_s = Coefficient of permeability of the saturated dredged material

k_o = Initial value of the coefficient of permeability of the saturated dredged material at the bottom of the deposit

k^* = Dimensionless coefficient of permeability

L = General differential operator

$L_{i,j}$ = General difference operator

m = Average rainfall in a certain 5-day period

M = Average number of rainy days in a given 5-day period

n = Number of intervals in which each integration domain has been divided

$n(i)$ = Number of rainy days in the i^{th} 5-day period

N = Number of soil layers

N = Number of tabulated years

$O\{\ \}$ = Error of order $\{\ \}$

p = Barometric pressure

p = Dimensionless total pressure

p = Probability of precipitation in any given day

$\bar{p}_o$ = Dimensionless initial effective pressure

q = Probability of no precipitation in any given day

Q_e = Amount of water entering an element of soil at a certain time

Q_ℓ = Amount of water leaving an element of soil at a certain time

r = Rainfall in a certain 5-day period

R = Gas constant for water vapor

R_c = Dayly precipitation

R_e = Total statistical rainfall during a given 5-day period

R_ℓ = Thickness ratio of the ℓ^{th} layer

s = Amount of consolidation

S = Degree of saturation

t = Time

t_t = Evaporation time threshold

T = Absolute temperature

T = Dimensionless time

u = Pore water pressure

u_a = Pore air pressure

u_w = Pore water pressure in the unsaturated soil

u^* = Pore water pressure before the end-of-step corrections

v = Wind velocity

$\underline{V}$ = Vector flow velocity

V_z = Velocity of water in the vertical direction

w = Weight water content

W_w = Total amount of water lost from the soil

x = Cartesian coordinate

x = Random number

y = Cartesian coordinate

y = Random number

z = Vertical cartesian coordinate

z = Random number

z_1 = Position of the water table at time t

z_2 = Position of the water table at time t + Δt

Z = Dimensionless vertical coordinate

$\alpha(z)$ = Coefficient of initial consolidation

β = Coefficient of transpiration

γ_w = Weight density of water

$\Gamma(z,t,x_1,x_2,\ldots)$ = Used as a general function describing a heterogeneous soil property

$\Gamma^*(t,x_1,x_2,\ldots)$ = Used as a general function describing a soil property at a given point

$\epsilon_{i,j}$ = Solution error

θ = Volumetric water content

θ^* = Volumetric water content before end-of-step corrections

θ_{cr} = Air-dry volumetric water content

κ = Unsaturated hydraulic conductivity

λ = Impedance factor

λ = Mesh factor ($\Delta t/\Delta z^2$)

μ = Empirical parameter for water retention functions

ν = Empirical parameter for water retention functions

ξ = Used as an intermediate time

ρ = Interface factor

σ = Total stress

$\bar{\sigma}$ = Effective stress

$\bar{\sigma}_o$ = Initial effective stress

τ = Used as an intermediate time

ϕ = Used to describe a general function

ϕ = Total water potential

χ = Bishop's coefficient for effective stress in unsaturated soils

ψ = Soil-water potential (also referred to as capillary potential or suction)

ψ_{cr} = Air-dry soil-water potential

ψ^* = Soil-water potential before end-of-step corrections

φ = Used as the approximation of a general function, ϕ

Ω = Volume

Ω_s = Volume of solids in an element of soil

Ω_T = Total volume of an element of soil

Ω_v = Volume of voids in an element of soil

Ω_w = Volume of water in an element of soil

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CHAPTER 1

INTRODUCTION

As a consequence of environmental considerations, most of the polluted dredgings from our nation's waterways and harbors are placed within diked containment areas that are constructed along the banks of rivers or the shores of lakes or to form small islands near the dredging sites. The dredgings are usually pumped hydraulically into these areas, where the coarser solids settle from suspension and the excess water with some suspended solids returns to the river or lake through an overflow weir. Some of the problems associated with this method of disposal are the high water content, low shear strength, and high compressibility of the materials, thereby rendering the landfill effectively unusable for long periods of time.

In order to understand more clearly and appreciate more fully the water content changes that take place in the sediments during and after the deposition process, a mathematical model was developed to describe the flow of water through the consolidating media. This model will (a) give the water content distribution in the fill at any time after the beginning of the deposition process, (b) predict the consolidation of the dredging sediments, and (c) help to evaluate the different techniques that can be used to accelerate the dewatering of the fill. It describes the flow of water through any type of soil at any degree of saturation and takes into consideration the consolidation process that occurs simultaneously.

The description of saturated and unsaturated zones separated by a moving boundary presents a number of mathematical and physical problems

that are due mainly to (a) the unknown location of the water table at any given time, (b) the dependence of the water table on weather conditions, and (c) the actual lack of knowledge regarding the volume change characteristics of partially saturated soils. In order to overcome these problems a step-by-step technique has been used in the discretization of the model and in its subsequent numerical solution by a digital computer. The general operating procedure for this step-by-step technique is to (a) assume that during a small time interval, Δt, a certain problem characteristic can be taken as constant, or linear, or continuous (depending on the nature of the problem), instead of variable, or nonlinear, or discontinuous, as is really the case, (b) solve the simplified problem during this time interval, Δt, and (c) adjust the solution to correct for the assumptions made in the first step.

Following a review in Chapter 2 of existing information on (a) flow through partially saturated soils, (b) volume change characteristics of partially saturated soils, (c) evapotranspiration, and (d) engineering properties of dredgings, a detailed development of the mathematical model is presented in Chapter 3. Chapter 4 gives the results of a very comprehensive laboratory and field test program that has been conducted on dredging wastes from the Toledo, Ohio area. The numerical solution of the mathematical model is described in Chapter 5, together with a detailed discussion of the convergence and stability techniques which have been employed. Chapter 6 is devoted to a study of the methods that can be used to accelerate the desiccation and consolidation process of the deposits and to predict the efficiency of each method by means of the mathematical model; such methods include the use of sand layers to improve drainage, increasing the evapotranspiration rate by means of plants, etc.

The summary and conclusions are presented in Chapter 7, and a flow chart and program description is included in the Appendix.

CHAPTER 2

BACKGROUND

The development of the mathematical model reported herein involves many aspects of soil behavior, ranging from those characteristics (consolidation, compressibility, etc.) traditionally considered by soil engineers to those (flow through partially saturated soils, evapotranspiration, etc.) normally treated by agronomists and soil scientists. Furthermore, the materials (namely, polluted maintenance dredgings that are deposited in diked containment areas to form landfills) to be handled are not well characterized in a general sense and their properties are not very well known.

The consolidation of a partially saturated medium leads to a variety of conflicting hypotheses. On one hand, soil mechanicians, who are interested mainly in deformations and volume change characteristics of soils subjected to external loads, have usually assumed full saturation, and the problem of defining concepts and coefficients for partially saturated media is one to which only incomplete and tentative answers exist; on the other hand, soil scientists, who have directed much effort to studying flow through partially saturated porous media, usually assume that such media possess a rigid structure and undergo no volume change during flow. The combination of these two approaches, which is the problem to be addressed in this study, therefore becomes very difficult because of the different assumptions that are involved. Included in this chapter is a review of some of the fundamentals from each approach, and an attempt is made to technically place the problem in perspective.

VOLUME CHANGE CHARACTERISTICS OF PARTIALLY SATURATED SOILS

In much of the work handled by soil engineers, the assumption of complete saturation is usually acceptable. Accordingly, relatively little research on partially saturated soils has been undertaken, and most of that which has been pursued has been directed towards shear strength properties. As a consequence, the volume change characteristics of partially saturated soils have been very much neglected by soil engineers.

The fundamental concept in predicting the behavior of fully saturated soils is the principle of effective stress. Although originally suggested for saturated soils by Terzaghi (1923, 1932), modified forms of effective stress principles have been proposed by Jennings (1957), Croney, Coleman, and Black (1958), Bishop (1959), Aitchison (1960), and others to take into account the presence of a third phase in the soil-water mixture. One very widely accepted equation (Bishop, 1959; Aitchison and Bishop, 1960) for describing the distribution of pore pressures in partially saturated soils is

$$\bar{\sigma} = (\sigma - u_a) + \chi(u_a - u_w), \qquad (2\text{-}1)$$

in which $\bar{\sigma}$ and σ are the effective and total stresses, u_a and u_w are the pore pressures in the air and water, and χ is an empirical parameter.

However, the nature of χ is not very well known, and the validity of Equation (2-1) has been questioned by Jennings and Burland (1962), Coleman (1962), Barden (1965), Matyas and Radhakrishna (1968), and others. Bishop and Blight (1963) have shown that χ depends not only on the soil structure, the degree of saturation, and the components $(\sigma - u_a)$ and $(u_a - u_w)$, but also on the stress path followed by both of the latter

components; this fact reduces greatly the practical value of Equation (2-1), even in its extended form

$$\bar{\sigma} = \sigma - u_a + f(u_a - u_w) \quad . \tag{2-2}$$

Jennings and Burland (1962), Coleman (1962), and research workers at MIT (1963) suggest that these two components should be treated separately, and Coleman, in particular, proposed a direct relationship between the volume change of soils and the applied stress increments; for one-dimensional compression this relationship is

$$\frac{\Delta\Omega}{\Omega} = C_1 \Delta(\sigma - u_a) + C_2 \Delta(u_a - u_w) \quad , \tag{2-3}$$

in which $\Delta\Omega/\Omega$ is the volumetric strain and C_1 and C_2 are compressibility coefficients. Barden, Madedor, and Sides (1969) have shown that C_1 and C_2 in Equation (2-3) depend on the existing state of stress and the soil characteristics, but, except for their dependence on the sign of the path derivative (that is, whether the stress is increasing or decreasing), these coefficients can be considered independent of stress path; the dependence on the sign of the stress path derivative is probably due to hysteresis between the saturation and desaturation processes. Despite the advances reported in the work described above, the compressibility of partially saturated soils is not yet clearly understood, and volume change relationships that are incorporated into mathematical models therefore require experimental verification.

FLOW THROUGH PARTIALLY SATURATED POROUS MEDIA

The study of flow through partially saturated soils has been of interest to soil scientists and agronomists in relation to the infiltration of rain water, the availability of water to plants, the limitation of water loss due to evapotranspiration, etc. The basic differential equation describing the flow of water through partially saturated soils (Richards, 1931) is

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (\kappa \nabla \phi), \tag{2-4}$$

where θ is the volumetric water content, t is time, κ is the unsaturated permeability (or conductivity), and ϕ is the total potential function. The concepts of "capillary potential" and "capillary conductivity" were first presented by Buckingham (1907), but it was not until many years later that these concepts were applied in the work of Gardner (1921) and Richards (1931). Klute (1952) applied these concepts to the theory developed by Childs (1936a, 1936b) and Childs and Collis-George (1950), who considered the soil water movement as a diffusion phenomenon, and he derived Equation (2-4) explicitly as

$$\frac{\partial \theta}{\partial t} = \nabla \cdot (D \nabla \theta) + \frac{\partial \kappa}{\partial z}, \tag{2-5}$$

in which the diffusivity, D, is defined as

$$D = \kappa \frac{\partial \psi}{\partial \theta}, \tag{2-6}$$

where ψ is the capillary potential. In the derivation of Equation (2-5) it is assumed that both κ and ψ are single-valued functions of θ and that the structure of the soil does not deform during flow. Equation (2-5) can

be expressed in terms of the capillary potential, Ψ, as

$$C \frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial z} \left(\kappa \frac{\partial \Psi}{\partial z} + \kappa \right), \tag{2-7}$$

in which the soil-water capacity, C, can be defined as

$$C = \frac{\partial \theta}{\partial \Psi} . \tag{2-8}$$

Equations (2-5) and (2-6) imply isothermal flow, and the term $\partial \kappa / \partial z$ should be eliminated if horizontal flow is considered. Much work has been done to obtain solutions to Equations (2-5) and (2-7), as related to different physical phenomena (drainage, infiltration, evapotranspiration, etc.). Many analytical solutions have been determined by Philip (1959, 1960a, 1960b), Childs (1967), Fritton, Kirkham, and Shaw (1970), Parlange (1971a, 1971b), and many others for specific functions $D(\theta)$, $\kappa(\theta)$, and $C(\psi)$. More numerous yet have been the numerical solutions (Philip, 1957, 1969; Hanks and Bowers, 1962; Whisler and Klute, 1965; Kobayashi, 1966; Chu Wang and Lakshminarayana, 1968; Jackson and Whisler, 1970; Staple, 1971; Watson and Whisler, 1972; Whisler, Watson, and Perrens, 1972; Casteleiro, 1972) to these equations. Although these studies dealt with both homogeneous and heterogeneous soils, none have apparently considered the variation in volume due to changes in the water content or any other cause.

EVAPORATION AND PLANT TRANSPIRATION FROM SOILS

The evaporation phenomena is a thermodynamic process in which the path of the Brownian motion of the molecules of water intersects the air-water interface, thus leaving the mass of water. Although few areas in hydrology have been as extensively investigated as evaporation, no exact

theoretical analysis is yet available.

Evaporation from Free Water Surfaces

As described by Linsley, Kohler, and Paulhus (1949), the rate of evaporation (that is, the rate at which water molecules leave the mass of water) depends on (a) the difference between the vapor pressure of the water and the vapor pressure of the air, (b) the air and water temperatures, (c) the wind speed, (d) the atmospheric pressure, and (e) the chemical composition of the water. Of the many empirical formulae used to predict evaporation, probably the best known is the one by Rohwer (1931), which can be written as

$$E_{pF} = C(1.465 - 0.0186p)(0.44 + 0.118v)(e_w - e_a), \qquad (2\text{-}9)$$

in which E_{pF} is daily evaporation in inches, p is the barometric pressure in inches of mercury, v is the wind speed in miles per hour, e_w is the vapor pressure at the water surface in inches of mercury, e_a is the average air vapor pressure in inches of mercury, and C is a coefficient which is equal to unity for evaporation from shallow ponds and equal to 0.771 for evaporation from reservoirs. Wisler and Brater (1959) note that the data needed to evaluate the evaporation by the Rohwer formula is readily available from any standard weather report.

Evaporation from Soil Surfaces

The evaporation from a soil surface does not differ greatly from the evaporation rate of a free water surface at the same temperature (Linsley, Kohler, and Paulhus, 1949), as long as the soil surface is saturated. When the soil becomes unsaturated, its hydraulic conductivity decreases rapidly (Philip, 1969), and the velocity at which the water may evaporate decreases sharply. There are two general methods to calculate

the evaporation rate from an unsaturated surface; the first (Staple, 1971) uses the Rohwer empirical formula represented by Equation (2-9), but replaces the vapor pressure at the water surface by the pressure of evaporation at a given temperature as a function of the capillary potential, and the second (Black, Gardner, and Thurtell, 1969; Bond and Willis, 1970, 1971) considers the existence of a "time threshold," t_t, such that

$$E_t = E_{pF}, \quad t \leq t_t \tag{2-10}$$

$$\int_{t_t}^{t} E_t = \int_{t_t}^{t} E_{pF} f(t - t_t)dt, \quad t > t_t, \tag{2-11}$$

in which E_t is the evaporation rate from the soil, E_{pF} is the evaporation rate from a free water surface under the same weather conditions, and t is time. The "time threshold," t_t, and the form of the function, $f(t - t_t)$, depend on the type of soil and the evaporation history, although this latter factor is often neglected (Hawks, Gardner, and Fairbourn, 1967). The experimental procedure is usually related to Equation (2-11), which represents the cumulative evaporation. Despite the fact that both of these methods are approximations (the second being purely experimental), they provide a useful means for handling the boundary condition at the surface of a soil subject to evaporation.

Transpiration

Transpiration is the process by which water is transferred from the soil to the atmosphere by plants. The transpiration rate, or transpiration potential, depends on (a) the available water, (b) the temperature, (c) the type of plants, (d) the stage of development of the plant, (e) the wind velocity, (f) the humidity of the atmosphere, (g) the depth of the

plant roots, (h) the relative position of the plant roots with respect to the water table, and (i) the intensity of the sunlight (Wisler and Brater, 1959). Due to the complex nature of the problem and the large ranges of variation for some of the variables, no theories or expressions (either empirical or analytical from a thermo-biological point of view) are apparently available to predict the transpiration rate as a function of the above stated variables. As a first order approximation, Baier (1967) suggests that the equation

$$E_p = (1 + \beta) E_t \tag{2-12}$$

can be used, in which E_p is the total evapotranspiration rate, E_t is the evaporation rate defined by Equations (2-10) or (2-11), and β is a parameter that depends on all the variables mentioned above (but β can be reasonably assumed constant for each type of crop or vegetation).

ENGINEERING PROPERTIES OF DREDGING WASTES

Although dredgings have frequentely been used for landfill, the motivation has usually been different and the dredged materials have generally been of better quality than the polluted materials considered in this study. As pointed out by Boyd, Saucier, Keeley, Montgomery, Brown, Mathis, and Guice (1972), it is virtually impossible to categorize in any simple way the large variety of polluted materials that are dredged from our nation's harbors and waterways, but several fairly comprehensive studies have been reported by the Corps of Engineers, Buffalo District (1969) and Philadelphia District (1969), Krizek, Karadi, and Hummel (1973), Reikenis, Elias, and Drabkowski (1974), and others. Within the context of the limitations inherent in studies of the type indicated above, it may be

reasonably inferred that most of the materials to which this model would apply are fine-grained silts and clays (average D_{10} of 0.002 mm and an uniformity coefficient of 6), high water content (100 to 140%), high plasticity (plasticity index of 20 to 50%), low strenght (3 to 10 kN/m^2), and low dry density (800 to 1000 kg/m^3). Very often such dredgings possess a moderate organic content (4 to 8%), and they will almost always contain a variety of different contaminants in various concentrations; however, the contaminants that are present do not seem to exert a significant influence on the engineering properties. In a given landfill most of the variation in properties with depth is due to the variation in water content; however, significant changes in material characteristics do occur in the horizontal direction with the coarser materials being found near the discharge pipe and the finer materials near the overflow weir. The need to understand more clearly the process of consolidation in these dredgings and to evaluate theoretically the effectiveness of various techniques for dewatering these materials supplies the impetus for the development of this mathematical model.

CHAPTER 3

THEORETICAL ANALYSIS

This chapter describes the theoretical formulation of the mathematical model that was developed to simulate, insofar as possible, the process of filling a diked containment area with dredged material and the consolidation of the deposits during and after the filling operation. Although the filling of the containment area may be accomplished in a number of ways (continuous pumping, intermittent pumping, direct dumping, etc.), thereby causing some variation in the associated sedimentation process, once the dredgings are in place the technique by which they were deposited becomes relatively unimportant with respect to the future processes that will occur in the layer of dredged material.

STATEMENT OF THE PROBLEM

The deposition of dredged material in a given containment area is usually accomplished over a period of several years. For purposes of this development, each year in which dredgings are placed in a particular area will be termed a cycle, and the incremental thickness of the sediment layer at the end of a cycle will be proportional to the amount of slurry placed in the site during that cycle. At the completion of the first cycle, there exists in the containment area (a) a fully saturated layer of sediments of variable thickness, H_1, with a certain void ratio distribution, $e(z)$, at each point (z is the vertical coordinate) and (b) an overlying layer of water of depth H_w, which is the vertical distance from the top of the sediment layer to the crest of the overflow weir that is constructed in the dike to return the excess water in the containment area to the river or lake.

After a certain period of time, the overlying water evaporates and the top of the layer begins to desiccate. Due to this desiccation and the possible drainage of water into the underlying foundation soil, the position of the water table will lower and the weight of the sediments comprising the layer will increase due to the loss of buoyancy by the soil situated above the phreatic surface, thereby causing the thickness of the layer to decrease. This process of desiccation, drainage, and consolidation continues until the beginning of the next cycle, at which time the site is again inundated.

During this second cycle, an additional sediment layer of thickness H_2 is deposited in the disposal area. If the process of deposition is relatively continuous, it can be assumed that the thickness of this layer will increase linearly, and thus the rate of pressure increase on the first layer will be constant. This process will be repeated as many times as necessary to fill the site. The principal phenomenon involved in this process is one of water flowing through a consolidating medium in which the location of the interface between the saturated and the partially saturated zones is unknown and dependent on the properties of the dredged material, weather conditions, the nature of the foundation material, and the particular stage of the cycle.

FLOW THROUGH HETEROGENEOUS POROUS MEDIA

Consider an elemental volume within a heterogeneous layer of dredged material through which there is a vertical movement of water, as shown in Figure 3-1. Suppose that (a) the center of the element is, at a certain time, at a distance z from the lower surface of the layer (z is the vertical coordinate, positive upward, and with its origin at the bottom

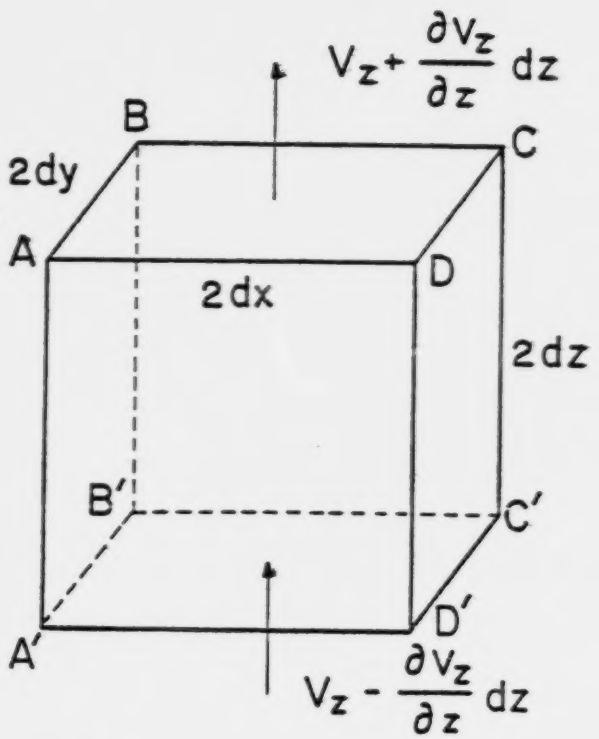

Figure 3-1. Elemental Volume of Consolidating Medium

of the layer) and (b) the velocity of the water at this point is V_z. If we assume that the velocity of deformation of the element can be neglected with respect to the velocity of the water (that is, small strains prevail), the amount of water entering the element through face A'B'C'D' can be expressed as

$$Q_{-} = \left(V_z - \frac{\partial V_z}{\partial z} dz\right) 4dxdy. \qquad (3\text{-}1)$$

Similarly, the amount of water leaving the element through face ABCD at a certain point in time is

$$Q_{\ell} = \left(V_z + \frac{\partial V_z}{\partial z} dz\right) 4dxdy. \qquad (3\text{-}2)$$

Thus, the volume of water stored in the elemental volume per unit time is

$$\frac{\partial \Omega_w}{\partial t} = - \frac{\partial V_z}{\partial z} \Omega_T , \tag{3-3}$$

where Ω_w is the volume of water and Ω_T is the total volume of the element. Letting Ω_v and Ω_s be the volumes of the voids and solids, respectively, we may write

$$\Omega_w = S \Omega_v = S e \Omega_s , \tag{3-4}$$

where e is the void ratio and S is the degree of saturation. Also, we have

$$\Omega_T = \Omega_v + \Omega_s = e \Omega_s + \Omega_s = (1 + e) \Omega_s . \tag{3-5}$$

The introduction of Equations (3-4) and (3-5) into Equation (3-3) yields

$$\frac{\partial}{\partial t} (S e \Omega_s) = - \frac{\partial V_z}{\partial z} (1 + e) \Omega_s . \tag{3-6}$$

Since we are assuming that the flow of solids can be neglected in comparison with the flow of water, Equation (3-6) becomes

$$\frac{\partial}{\partial t} (Se) = - \frac{\partial V_z}{\partial z} (1 + e), \tag{3-7}$$

where the degree of saturation, S, is defined as

$$S = \frac{\Omega_w}{\Omega_v} = \frac{\Omega_w}{e\Omega_s} . \tag{3-8}$$

Use of Equation (3-5) allows Equation (3-8) to be written as

$$S = \frac{1+e}{e} \frac{\Omega_w}{\Omega_T} . \tag{3-9}$$

The term Ω_w/Ω_T is called the volumetric water content and it is usually represented by θ. Thus, Equation (3-7) may finally be written as

$$\frac{\partial}{\partial t}\left[(1 + e)\ \theta\right] = -\frac{\partial V_z}{\partial z}\ (1 + e). \tag{3-10}$$

The flow velocity of water through a saturated porous medium may be expressed from Darcy's law as

$$\underline{V} = -k \nabla \phi, \tag{3-11}$$

where $\underline{V}$ is the vector flow velocity, k is the coefficient of permeability of the dredgings, and ϕ is the total potential. Equation (3-11) holds for small values of Reynold's number (that is, for slow flow). Childs and Collis-George (1950) confirmed experimentally that Equation (3-11) also holds for partially saturated soils as long as the water channels through the voids can be assumed continuous, as is apparently the case even for values of the water content near the air-dry value. When applied to non-saturated flow, k is usually termed the hydraulic conductivity and it is a function of the water content (decreasing rapidly as the water content decreases). Throughout this study, however, k will be called the permeability, which is a commonly used engineering term. For one-dimensional vertical flow, Equation (3-11) reduces to

$$V_z = -k \frac{\partial \phi}{\partial z}. \tag{3-12}$$

By introducing Equation (3-12) into Equation (3-10), we get

$$\frac{\partial}{\partial t}\left[(1 + e)\ \phi\right] = \frac{\partial}{\partial z}\left(k \frac{\partial \phi}{\partial z}\right)(1 + e), \tag{3-13}$$

or

$$\frac{1}{1+e}\left[\theta\,\frac{\partial e}{\partial t} + (1+e)\,\frac{\partial \theta}{\partial t}\right] = \frac{\partial}{\partial z}\left(k\,\frac{\partial \phi}{\partial z}\right). \tag{3-14}$$

Equation (3-14) describes the one-dimensional (vertical) flow of water through a porous medium, regardless of the degree of saturation and the type of potential (external load, suction, etc.), and, within the framework of small strain theory, it takes into consideration the possible consolidation of the layer. In order to transform Equation (3-14) into a one-variable differential equation, two additional relationships are needed; these are readily available in the case of a saturated medium, but the case of partially saturated flow presents a formidable challenge.

Saturated Flow

In the case of saturated flow, the total isothermal potential is

$$\phi = \frac{u}{\gamma_w}, \tag{3-15}$$

in which u is the excess pore water pressure and γ_w is the weight density of water. From Equation (3-9) we get (for S=1)

$$\theta = \frac{e}{1+e}, \tag{3-16}$$

which, together with the assumption that water is incompressible, can be combined with Equation (3-14) to obtain

$$\frac{1}{1+e}\left[\frac{e}{1+e}\,\frac{\partial e}{\partial t} + \frac{1+e}{(1+e)^2}\,\frac{\partial e}{\partial t}\right] = \frac{1}{\gamma_w}\,\frac{\partial}{\partial z}\left[k\,\frac{\partial u}{\partial z}\right], \tag{3-17}$$

which further reduces to

$$\frac{\gamma_w}{1+e}\frac{\partial e}{\partial t} = \frac{\partial}{\partial z}\left[k\,\frac{\partial u}{\partial z}\right] . \qquad (3\text{-}18)$$

Krizek and Salem (1974) have shown that the volume change characteristics of these dredgings, as well as most other soils, may be expressed as

$$e = e_o - C_c \log \frac{\bar{\sigma}_o+\bar{\sigma}}{\bar{\sigma}_o} , \qquad (3\text{-}19)$$

where e_o is the initial void ratio, C_c is the compression index, $\bar{\sigma}_o$ is the initial effective stress, and $\bar{\sigma}$ is the increase in effective stress. The derivative of Equation (3-19) with respect to time is

$$\frac{\partial e}{\partial t} = -\frac{0.4343\ C_c}{(\bar{\sigma}_o+\bar{\sigma})}\frac{\partial(\bar{\sigma})}{\partial t} . \qquad (3\text{-}20)$$

From the principle of effective stress we may write

$$\bar{\sigma} = \sigma - u , \qquad (3\text{-}21)$$

where σ is the total applied stress, and the combination of Equations (3-20) and (3-21) gives

$$\frac{\partial e}{\partial t} = -\frac{0.4343\ C_c}{(\bar{\sigma}_o+\sigma-u)}\left[\frac{\partial \sigma}{\partial t} - \frac{\partial u}{\partial t}\right] , \qquad (3\text{-}22)$$

which, together with Equation (3-18), yields

$$\frac{0.4343\ C_c \gamma_w}{(1+e)(\bar{\sigma}_o+\sigma-u)}\left[\frac{\partial u}{\partial t} - \frac{\partial \sigma}{\partial t}\right] = \frac{\partial}{\partial z}\left[k\,\frac{\partial u}{\partial z}\right] . \qquad (3\text{-}23)$$

In general, e, C_c, k, $\bar{\sigma}_o$, and σ will be functions of z, as well as t. The time-dependence of C_c and k is due to the changes that take place in e during the consolidation process, and $\bar{\sigma}_o$ reflects the previous history of the fill. The increase in the total pressure, σ, is due to the lowering

of the water table, but, since the position of the water table can not be predicted beforehand, the term $\partial\sigma/\partial t$ is also unknown.

Partially Saturated Flow

In the case of partially saturated flow, the total isothermal potential is

$$\phi = \psi + z \tag{3-24}$$

where ψ is the capillary potential and z is the vertical coordinate. In fine-grained soils the air entry value (the water pressure at which the soil becomes unsaturated) is very small (Philip, 1969), and it will therefore be taken as zero throughout this analysis; this means that, for the soil to become unsaturated, the pore water pressure must fall below zero. This negative pore water pressure is called capillarity, capillary pressure, capillary potential, or suction, and it is represented by ψ in Equation (3-24). In general, the relationship between ψ and θ is not unique, but manifests a hysteretic effect between the wetting and drying processes. However, in this work a unique relationship that is representative of the drying process will be assumed; this is justified on the bases that (a) although infiltration and wetting will certainly occur periodically during the time span under consideration, the wetting periods will be relatively short compared to the drying periods and they will be impeded by the dry crust that forms at the surface of the layer and (b) once the dredgings become unsaturated, the water content distribution has almost no effect on the consolidation of the layer and the associated small errors (usually less than 10% variation in volumetric water content; Philip, 1969) can be neglected with little loss in generality. Substituting Equation (3-24) in Equation (3-14), gives

$$\frac{1}{1+e}\left[\theta\frac{\partial e}{\partial t} + (1+e)\frac{\partial\theta}{\partial t}\right] = \frac{\partial}{\partial z}\left[k\frac{\partial\Psi}{\partial z} + k\right], \tag{3-25}$$

in which the permeability, k, is a function of both z and θ or Ψ, since $\theta=f(\Psi)$. The derivative of θ with respect to time can be written as

$$\frac{\partial\theta}{\partial t} = \frac{\partial\theta}{\partial\Psi}\frac{\partial\Psi}{\partial t} = C\frac{\partial\Psi}{\partial t}, \tag{3-26}$$

where C, which is the specific water capacity of the dredgings, is a function of z and Ψ and can be derived from water retention data. Finally, the equation describing the one-dimensional vertical flow of water through a partially saturated consolidating medium is

$$\frac{1}{1+e}\left[\theta\frac{\partial e}{\partial t} + C(1+e)\frac{\partial\Psi}{\partial t}\right] = \frac{\partial}{\partial z}\left[k\frac{\partial\Psi}{\partial z} + k\right]. \tag{3-27}$$

The term $\theta(\partial e/\partial t)$ results from the fact that the soil is undergoing a process of consolidation which, in general, can not be expressed as a function of Ψ and its derivative. Although various expressions have been suggested to describe the volume change characteristics of partially saturated soils, none has proven to be simple or general enough for widespread use. Coleman (1962) proposed a relationship of the form

$$\frac{d\Omega}{\Omega} = C_1\, d(\sigma - u_a) + C_2\, d(u_a - u_w), \tag{3-28}$$

in which Ω is volume, σ is total stress, u_a is pore air pressure, u_w is pore water pressure, and C_1 and C_2 are compressibilities. Equation (3-28) has been experimentally tested by Barden, Madedor, and Sides (1969), but no general conclusions were advanced to justify its use; however, their results seem to indicate that, unlike Bishop's coefficient, χ, C_1 and C_2 are fairly independent of the stress paths of $(\sigma-u_a)$ and (u_a-u_w). In any

case the equipment and experimental procedures needed to obtain the compressibilities C_1 and C_2 are complicated (MIT, 1963), and such efforts will rarely be justified in situations such as those addressed in this work. Furthermore, under normal field conditions the water table remains relatively high and only the top of the layer becomes desiccated, thereby minimizing the error that might be introduced due to inadequate knowledge regarding the volume change characteristics of partially saturated soils.

INITIAL CONDITIONS

By the end of the i^{th} cycle, a layer of sediments of thickness H_i will have been deposited in the containment area, and the total stress, $\sigma_o(z)$, at a point located a distance z from the bottom of the layer will be

$$\sigma_o(z) = \gamma_w \int_z^{H_i} \frac{G_s - 1}{1+e} \, dz \, , \qquad (3\text{-}29)$$

where G_s is the specific gravity of the solids. This stress will have produced a certain consolidation in the layer, the amount of which can not be explicitly determined; however, the actual amount of consolidation is not as important to know as the distribution of effective stress throughout the layer. In general, the effective stress, $\overline{\sigma}_o(z)$, at a point can be expressed as

$$\overline{\sigma}_o(z) = \alpha(z) \, \sigma_o(z) \, , \qquad (3\text{-}30)$$

where $\sigma_o(z)$ is the total stress given by Equation (3-29) and $\alpha(z)$ is a coefficient less than one. The value of $\alpha(z)$ at each point will depend on (a) the rate at which the thickness of the sediment layer increases, (b) the characteristics of the dredged material, and (c) the drainage conditions

at the bottom of the layer. Although the exact value of $\alpha(z)$ is not known, it can be reasonably assumed that, due to the extreme slowness of the process, $\alpha(z)$ will be very close to one for most of the layer. For example, dredged material was deposited in Penn 7 to a depth of about 2 meters during a period (first cycle) of 154 days from August 28, 1972, to December 19, 1972; this represents an average deposition of about 1.3 cm/day and an average increase in total stress of about 0.5 gm/cm^2 per day (assuming a specific gravity of solids of 2.70 and an initial void ratio of 3.0). In most cases $\alpha(z)$ will be assumed equal to one for $0 \leq z \leq H_i$, but the impact of this assumption on the final solution will be quantified in Chapter 6.

BOUNDARY CONDITIONS

One of the easiest ways to control the rates of desiccation and consolidation is through modifications to the boundary conditions. Accordingly, the mathematical formulation for various boundary conditions will be established, and the relative influence of different conditions on the final solution will be evaluated in Chapter 6.

Lower Boundary

Three types of boundary conditions may exist at the lower boundary of the dredgings layer, depending on the nature of the underlying ground; these are (a) free drainage, for which

$$u(o,t) = 0; \quad t \geq 0; \tag{3-31}$$

(b) an impervious boundary, where

$$\frac{\partial u}{\partial z}\Big|_{o,t} = 0; \quad t \geq 0 \tag{3-32}$$

for complete saturation, and

$$\left.\frac{\partial v}{\partial z}\right|_{o,t} + 1 = 0; \quad t \geq 0 \tag{3-33}$$

for partial saturation; and (c) impeded drainage. In the latter case the only exact solution involves extending the domain of integration to the foundation soil, but this is not necessary for the problem considered herein because the underlying soil is much more rigid than the layer of dredge spoil and it can be assumed incompressible without significant loss in generality. If it is further assumed that the underlying soil is homogeneous, its consolidation behavior can be approximated by

$$c_v \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \quad , \tag{3-34}$$

where c_v is the coefficient of consolidation. However, since the layer is assumed to be incompressible, $c_v = \infty$ and therefore $\partial^2 u/\partial z^2 = 0$. On the other hand, the condition of continuity at the boundary may be written

$$k_c \left.\frac{\partial u}{\partial z}\right|_c = k_f \left.\frac{\partial u}{\partial z}\right|_f \quad , \tag{3-35}$$

where c designates the consolidating layer and f the foundation material. Suppose now that there is a free draining boundary at a distance H_f from the top of the foundation soil, as shown in Figure 3-2. Integrating $\partial^2 u/\partial z^2 = 0$ over the domain $-H_f \leq z \leq 0$ and taking into consideration that $u(-H_f) = 0$, we obtain

$$u(z) = u(o,t) \frac{H_f + z}{H_f} \quad . \tag{3-36}$$

Since the excess pore pressure, u, does not depend on time in the domain

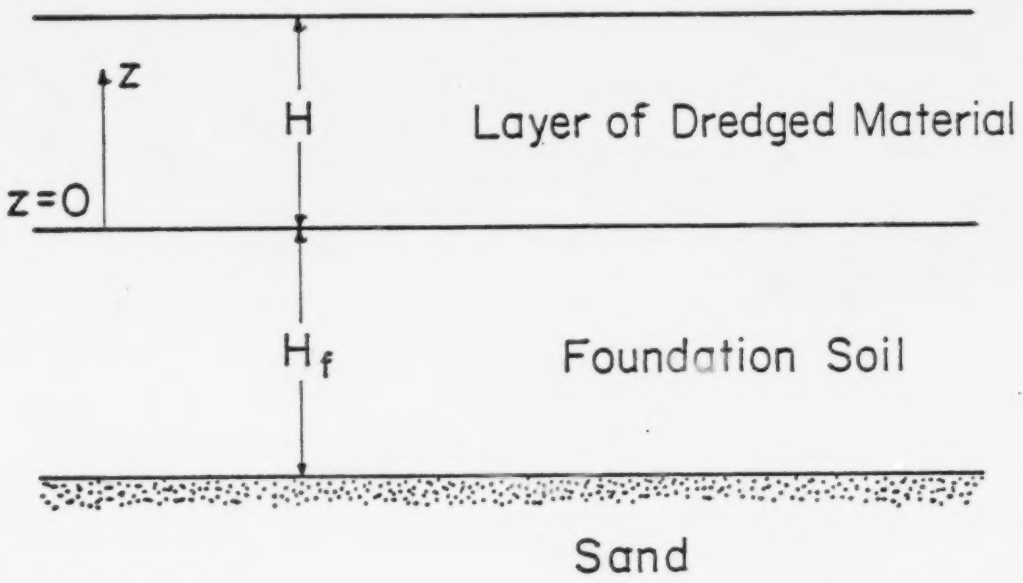

Figure 3-2. Profile of Layered System Under Consideration

$-H_f \leq z \leq 0$, the foregoing equation may be written in terms of total derivatives. Introducing Equation (3-36) into Equation (3-35) yields

$$k_c \left.\frac{\partial u}{\partial z}\right|_{o,t} = k_f \frac{\partial}{\partial z}\left[u(o,t) \frac{H_f + z}{H_f} \right], \tag{3-37}$$

which can be reduced to

$$k_c \left.\frac{\partial u}{\partial z}\right|_{o,t} - \frac{k_f}{H_f} u(o,t) = 0. \tag{3-38}$$

Note that, if the nonconsolidating foundation were underlain by an impervious boundary instead of a sand layer, the boundary condition at z=0 would be the same as that for an impervious boundary.

The likelihood that Equation (3-38) will adequately characterize a condition of impeded drainage in an actual field situation is rather small, since the foundation soil will usually be heterogeneous and compressible,

and its void ratio-effective stress relationship may be nonlinear. Nevertheless, if the impedance factor, λ, is represented by

$$\lambda = \frac{k_f}{k_c} \frac{1}{H_f} \quad , \tag{3-39}$$

the effect of variations in λ on the rate of pore pressure dissipation can be studied.

In addition to the preceding three natural boundary conditions, artificial or "forced" drainage can be introduced at the bottom of the dredgings layer. For example, a partial vacuum may be applied to a sand blanket underlying the layer of sediments; if the vacuum were maintained constant, the boundary condition would be $u=-\gamma_w h_v$, where γ_w is the weight density of water and h_v is the extent of the vacuum. Although provision for such a forced drainage condition has been incorporated in the mathematical model, this situation is probably too elaborate for the type of problem under consideration.

Upper Boundary

The loss of water through the top surface of the dredgings layer is caused by evapotranspiration. Evaporation is a very complicated thermodynamic process by which the free energy of the water molecules moving in a mass of water is increased beyond the energy of the molecules moving in the air near the surface of the water. As a consequence of this energy difference, the number of molecules leaving the water is greater than the number entering it, and thus the quantity of water in the mass decreases while the air increases its water vapor saturation. In order to circumvent the difficulty associated with formulating an analytical theory to characterize water molecules moving in two different media at random velocities along random paths and liberating random amounts of energy, a variety

of empirical formulae have been proposed to predict the evaporation rate from a free water surface. In the equation proposed by Rohwer (1931) and used in this study, the daily evaporation in inches, E_{pF}, is calculated from the relationship

$$E_{pF} = (1.465 - 0.0186\ p)(0.44 + 0.118\ v)(e_w - e_a), \tag{3-40}$$

where p is the barometric pressure in inches of mercury, v is the mean wind speed in miles per hour, e_w is the vapor pressure at the water surface, and e_a is the air vapor pressure (both e_w and e_a are expressed in inches of mercury). This formula has given good results (Wisler and Brater, 1959), and it has the advantage that the data involved in the calculations can be obtained directly from standard weather station records (in this case records were obtained from the U.S. Department of Commerce, National Oceanic and Atmospheric Administration), thereby obviating the need for additional experimental work.

Equation (3-40) represents the evaporation rate from a free water surface or from the surface of a saturated soil (Nerpin, Pakshina, and Bondarenko, 1966). However, when the soil begins to desiccate, the vapor pressure at a free water surface, e_w, must be replaced by a modified vapor pressure, e_s, which takes into account the surface tension caused by the capillary forces. Linsley, Kohler, and Paulhus (1949) suggest that the relationship between e_w and e_s can be expressed as

$$e_s = e_w \exp\left[\frac{\Psi}{RT}\right], \tag{3-41}$$

where Ψ is the capillary potential at the surface of evaporation, R is the gas constant for water vapor, and T the absolute temperature in degrees

Kelvin. A similar relationship was obtained by Benetin and Cervenkova (1966). Thus, the evaporation from a bare soil (that is, one with no surface vegetation) can be expressed by

$$E_T = (1.465 - 0.0186\ p)(0.44 + 0.118\ v)\left[e_w \cdot \exp\left(\frac{\psi}{RT}\right) - e_a\right]. \quad (3\text{-}42)$$

In addition to evaporation, the dredgings also lose water due to transpiration by plants, and this phenomenon is even more complicated than that of evaporation. As pointed out in Chapter 2, there has been extensive experimental work in this field (particularly with regard to commercial crops, like corn, wheat, cotton, etc.), but the number and nature of the variables involved virtually preclude any general empirical or analytical solution. As a first order approximation, Baier (1967) suggests the use of the expression

$$E_p = (1 + \beta)\ E_T, \quad (3\text{-}43)$$

in which E_p is the total evapotranspiration rate, E_T is the evaporation rate as calculated by Equation (3-42), and β is a coefficient that must be obtained experimentally for each type of plant. However, Equation (3-43) must be used with caution, because the osmotic interaction of the root-soil system is too complicated to be explained by such a simple expression, which, in addition, neglects a number of other important factors, such as the age of the plants, density of the root system, radiation hours, wilting characteristics, etc. The effect of the coefficient β on the flow model will be studied in Chapter 6, but only as a first approximation.

If the temperature is below freezing, the results of Equation (3-42) should be multiplied by a factor of 0.8 (Croft, 1944). Furthermore, Equation (3-42) represents only the evaporation from the soil surface without taking into consideration the precipitation that may have occurred. If R_c is the precipitation (in inches) recorded on a certain day and E_p is the evapotranspiration rate for the same day, the total amount of water (in inches), W_w, lost from the soil is

$$W_w = E_p - R_c . \tag{3-44}$$

Since this loss must be equal to the flow through the upper surface of the soil, the upper boundary condition, if W_w is positive, will be

$$k\left(\frac{\partial \Psi}{\partial z} + 1\right) = -W_w \tag{3-45}$$

If W_w is negative (that is, the precipitation is greater than the evaporation), an infiltration situation prevails and the boundary condition is

$$\Psi = 0 . \tag{3-46}$$

Staple (1971) has shown that, for a certain capillary potential, Ψ_{cr}, the upper surface becomes stationary (Ψ=constant), regardless of the value of W_w (as long as it is positive). This is the case because at this critical potential the water channels become discontinuous, Darcy's law no longer applies, and no water transfer can take place between the upper surface and the air (of course, the transfer of water vapor from deeper surfaces will continue). Staple determined that this threshold value of Ψ is equal to the air-dry capillarity potential. Therefore, once the capillarity potential becomes less than or equal to the critical potential, Ψ_{cr}, the boundary

condition at the upper surface is

$$\Psi = \Psi_{cr} \quad . \tag{3-47}$$

In order to apply representative, but nonspecific, weather conditions to Equations (3-42) and (3-44), a simple statistical model has been developed. Weather data for a number of years were tabulated and averages for every five days were calculated (index periods of five days or a week are frequently used in meteorology). The calculated parameters are the average temperature, the average dewpoint, the average wind speed, the average barometric pressure, the total precipitation, and the number of days of measurable precipitation. From the average temperature and the average dewpoint, e_w and e_a can be calculated, respectively (as for example, from the tables in Wisler and Brater, 1959). Then, a uniformly distributed random number x, such that $0 \leq x \leq 1$, is generated by a Monte Carlo technique. If N years have been tabulated, the data corresponding to the i^{th} year will be selected if

$$\frac{i-1}{N} \leq x < \frac{i}{N} \; ; \quad i = 1, N. \tag{3-48}$$

With these data the evapotranspiration rate can be calculated from Equations (3-42) and (3-43).

To calculate the precipitation, a different approach is used. If n(i) are the number of rainy days in the i^{th} five-day period corresponding to a certain time of the year (one period for each tabulated year), the average number of rainy days in the five-day period corresponding to that time of the year is

$$M = \frac{1}{N}\sum_{i=1}^{N} n(i)\,, \tag{3-49}$$

and the probability of measurable precipitation in any one day of the group is

$$p = \frac{M}{5}. \tag{3-50}$$

Finally, the probability of no precipitation in any one day is

$$q = 1 - p\,. \tag{3-51}$$

If we assume that the event "no rain in the i^{th} day" is independent of the event "no rain in the j^{th} day," the probability of the time group being without precipitation is

$$P = q^5 = (1 - p)^5\,. \tag{3-52}$$

If another random number y, generated as before, is such that $0 \leq y < P$, the five-day period will be taken as "zero precipitation." If on the other hand, $y \geq P$, the period will be taken as "rainy."

Let m be the average precipitation in a rainy period for the N tabulated years. Benjamin and Cornell (1970) suggest that the probability mass function of the rainfall in any period, given that this period is rainy, can be expressed as

$$f_R(r) = \frac{1}{m}\, e^{-r/m}; \quad r \geq 0\,, \tag{3-53}$$

in which r is the rainfall. To use Equation (3-53) it must be assumed that the distribution is identical in different periods. The cumulative

distribution function corresponding to Equation (3-53) is

$$F_R(r) = 1 - e^{-r/m} . \tag{3-54}$$

If this function is uniformly distributed, the corresponding values of r will adjust to Equation (3-53) and give intervals of equal probability. Generation of another random number z, as before, yields

$$z = 1 - e^{-r/m} , \tag{3-55}$$

which may be written as

$$r = -m \, \ell n(1 - z) . \tag{3-56}$$

However, this value represents the total precipitation, R_e, during the period whereupon we can write

$$R_e = -m \, \ell n(1 - z) . \tag{3-57}$$

The use of Equations (3-42), (3-43), (3-44), and (3-57) allows the boundary condition at the upper surface of the dredgings layer to be established.

Boundary Between Different Layers

After the i^{th} cycle is complete, there will be i different layers in the containment area. In general, these layers will have different characteristics, which means that some continuity condition will have to be established at their interfaces. In addition to the fact that the water pressure must be unique at the interfaces of contiguous layers, continuity condition for flow can be written as

$$\left(k \frac{\partial u}{\partial z}\right)_{i-1,t} = \left(k \frac{\partial u}{\partial z}\right)_{i,t} , \tag{3-58}$$

if the soil is saturated at this point, or as

$$\left[k\left(\frac{\partial\psi}{\partial z}+1\right)\right]_{i-1,t}=\left[k\left(\frac{\partial\psi}{\partial z}+1\right)\right]_{i,t}, \tag{3-59}$$

if the soil is partially saturated. The numerical technique used to introduce the continuity condition into the corresponding field equation will be explained in Chapter 5.

STEP-BY-STEP SOLUTION

To summarize, the differential equations governing the flow process are

$$\frac{0.4343\ C_c\gamma_w}{(1+e)(\bar{\sigma}_o+\sigma-u)}\left[\frac{\partial u}{\partial t}-\frac{\partial\sigma}{\partial t}\right]=\frac{\partial}{\partial z}\left(k\frac{\partial u}{\partial z}\right) \tag{3-60}$$

and

$$\frac{1}{1+e}\left[\theta\frac{\partial e}{\partial t}+C(1+e)\frac{\partial\psi}{\partial t}\right]=\frac{\partial}{\partial z}\left(k\frac{\partial\psi}{\partial z}+k\right) \tag{3-61}$$

for saturated and partially saturated phases, respectively. Both of these equations have a term which is not generally known. In the case of a saturated soil, the term $\partial\sigma/\partial t$ can not be calculated, because the future behavior of the water table is not known. In the case corresponding to partially saturated flow, the term $\partial e/\partial t$ is unknown, because no single relationship can be established between the void ratio and the capillary potential. Nevertheless, the procedures of numerical calculus allow these difficulties to be overcome through an approximation technique known as the step-by-step method.

Suppose that at a certain time, t, the water table is at a position z_1 and the distribution of pore water pressure is $u(z,t)$ below the water table (saturated zone) and $\psi(z,t)$ above the water table (unsaturated zone); furthermore, the void ratio and water content distributions will be $e(z,t)$ and $\theta(z,t)$, respectively. Next, assume that during the next small time

interval, Δt, the total pressure will not increase and the unsaturated zone is incompressible; then, Equations (3-60) and (3-61) reduce to

$$\frac{0.4343\ C_c \gamma_w}{(1+e)(\bar{\sigma}_o+\sigma-u)} \frac{\partial u}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial u}{\partial z}\right) \tag{3-62}$$

and

$$C \frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial \Psi}{\partial z} + k\right). \tag{3-63}$$

These equations can now be solved in conjunction with the previously established boundary conditions. If we assume that the new position of the water table after Δt is z_2, the pore pressure distribution will be

$$u^*(z,\ t + \Delta t);\quad 0 \le z \le z_2\ , \tag{3-64a}$$

$$\Psi^*(z,\ t + \Delta t);\quad z_2 \le z \le H\ . \tag{3-64b}$$

The decrease in pore pressure in the saturated zone is due to an increase in the effective pressure, given by

$$\Delta\bar{\sigma}(z) = u(z,\ t)\ -\ u^*(z,\ t + \Delta t)\ , \tag{3-65}$$

which will cause a decrease in the void ratio. The new void ratio distribution will be

$$e(z,\ t + \Delta t) = e(z,\ t)\ -\ C_c(z)\ \log \frac{\bar{\sigma}_o(z)+\Delta\bar{\sigma}(z)}{\bar{\sigma}_o(z)}\ . \tag{3-66}$$

In the unsaturated zone (except in the portion $z_2 \le z \le z_1$) the application of Coleman's theory gives the variation in volume as

$$\frac{\Delta\Omega}{\Omega} = C_1\ \Delta(\sigma\ -\ u_a)\ +\ C_2\ \Delta(u_a\ -\ u_w)\ . \tag{3-67}$$

As the problem under consideration involves desiccation from the top surface, the pore air pressure must be equal to the atmospheric pressure, since the only way air can get to the dredgings layer is through the top boundary and all air channels must therefore be connected with the atmosphere. Then, Equation (3-67) becomes

$$\frac{\Delta\Omega}{\Omega} = C_1 \, \Delta\sigma - C_2 \, \Delta \, u_w \, . \tag{3-68}$$

However, since only one-dimensional deformation is being considered, we have

$$\frac{\Delta\Omega}{\Omega} = \frac{\Delta e(z)}{1+e(z,t)} \, , \tag{3-69}$$

and since the total stress is assumed constant during the interval, Equation (3-68) can be reduced to

$$\Delta e(z) = - \left[1 + e(z, \, t) \right] C_2 \, \Delta \, u_w \, , \tag{3-70}$$

which, with the use of

$$\Delta \, u_w = \gamma_w \left[\Psi(z, \, t) - \Psi^*(z, \, t + \Delta t) \right], \tag{3-71}$$

allows the variation in the void ratio in the zone $z_1 \leq z \leq H$ to be expressed as

$$\Delta e(z) = -\gamma_w \, C_2 \left[1 + e(z, \, t) \right] \left[\Psi(z, \, t) - \Psi^*(z, \, t + \Delta t \right] \, , \tag{3-72}$$

and the new void ratio becomes

$$e(z, \, t + \Delta t) = e(z, \, t) + \Delta e(z) \, . \tag{3-73}$$

In the zone $z_2 \leq z \leq z_1$ the soil has become unsaturated; accordingly, all of the initial excess pore water pressure has dissipated and the volume reduction due to this dissipation is

$$\Delta e_1^*(z) = -C_c(z) \log \frac{\bar{\sigma}_o + u(z,t)}{\bar{\sigma}_o} . \tag{3-74}$$

In addition, the capillary potential distribution causes another volume reduction, which may be expressed as

$$\Delta e_2^*(z) = -\gamma_w C_2 \left[1 + e(z, t) + \Delta e_1^*(z)\right]\left[-\psi^*(z, t + \Delta t)\right], \tag{3-75}$$

and the new void ratio in the zone $z_2 \leq z \leq z_1$ becomes

$$e^*(z, t + \Delta t) = e(z, t) + \Delta e_1^*(z) + \Delta e_2^*(z) . \tag{3-76}$$

Now, we must correct the first assumption, because the total stress in the region $0 \leq z \leq z_1$ does indeed change due to the loss of buoyancy in the zone $z_2 \leq z \leq z_1$; this stress increase can be written as

$$\Delta\sigma_1 = \int_{z_2}^{z_1} \left[\theta^*(z, t + \Delta t) + \frac{1}{1+e^*(z,t+\Delta t)}\right] dz; \quad 0 \leq z \leq z_2 , \tag{3-77}$$

$$\Delta\sigma_2 = \int_{z}^{z_1} \left[\theta^*(z, t + \Delta t) + \frac{1}{1+e^*(z,t+\Delta t)}\right] dz; \quad z_2 \leq z \leq z_1 , \tag{3-78}$$

where $\theta^*(z,t+\Delta t)$ has been previously related to the soil-water potential functions corresponding to $\psi^*(z,t+\Delta t)$, $z_2 \leq z \leq z_1$, and $e^*(z,t+\Delta t)$ has been evaluated from Equation (3-76). We shall now assume that the increase in the total stress takes place at the end of the time step and is instantaneous. In this case the increase in the total stress in the saturated zone will produce a pore water pressure increase of the same magnitude,

namely

$$\sigma(z,\ t + \Delta t) = \sigma(z,\ t) + \Delta\sigma_1;\quad 0 \le z \le z_2 \tag{3-79a}$$

$$u(z,\ t + \Delta t) = u^*(z,\ t + \Delta t) + \Delta\sigma_1;\quad 0 \le z \le z_2. \tag{3-79b}$$

In the unsaturated zone the increase in the total stress will, in general, produce an increase in both the pore water and the pore air pressures. In this case, however, since the air in the pores is assumed to remair at atmospheric pressure, the result will be the same as in the saturated zone, namely

$$\sigma(z,\ t + \Delta t) = \sigma(z,\ t) + \Delta\sigma_2;\quad z_2 \le z \le z_1 \tag{3-80a}$$

$$\Psi(z,\ t + \Delta t) = \Psi^*(z,\ t + \Delta t) + \frac{\Delta\sigma_2}{\gamma_w};\quad z_2 \le z \le z_1. \tag{3-80b}$$

Hence, the new pore water head distribution, $\Psi(z,t+\Delta t)$, $z_2 \le z \le z_1$, will not correspond to the previously obtained water content distribution $\theta^*(z,t+\Delta t)$, $z_2 \le z \le z_1$, and a new distribution must be obtained for $\Psi(z,t+\Delta t)$; this is accomplished by using the soil-water potential functions. In order to achieve an instantaneous reduction of volumetric water content, the void ratio must change. Use of the definition for water content, $\theta=\Omega_w/\Omega_T$, and the relation $\Omega_T=1+e$ allows us to write $\Omega_w=\theta(1+e)$. Due to the instantaneous increase in the pore water potential, no water will enter the volume of soil; therefore, Ω_w will remain constant. If $\theta(z,t+\Delta t)$ and $e(z,t+\Delta t)$, $z_1 \le z \le z_2$, are the new water content and void ratio, respectively, we may write

$$\theta(z,\ t + \Delta t)\Big[1 + e(z,\ t + \Delta t)\Big] = \theta^*(z,\ t + \Delta t)\Big[1 + e(z,\ t + \Delta t)\Big], \tag{3-81}$$

where $e^*(z,t+\Delta t)$ is obtained from Equation (3-76). Thus, the final void

ratio distribution in the zone $z_2 \leq z \leq z_1$ is

$$e(z,\ t + \Delta t) = \left[1 + e^*(z,\ t + \Delta t)\right] \frac{\theta^*(z,t+\Delta t)}{\theta(z,t+\Delta t)} - 1. \qquad (3\text{-}82)$$

Once the water pressures and the void ratio and water content distributions are known at $t+\Delta t$, a new time step can be started and the procedure can be repeated. The amount of consolidation, Δs, after each time step will be calculated as

$$\Delta s = \int_0^H \frac{e(z,t) - e(z,t+\Delta t)}{1+e(z,t)}\, dz\,. \qquad (3\text{-}83)$$

To the authors' knowledge, no method is available to estimate the inherent errors in a step-by-step procedure such as the one described. However, the processes involved are relatively slow, and, if the time increment is sufficiently small, the error will hopefully be of less importance than the errors associated with handling and interpolating the data. The consolidation that occurs during a new cycle will be calculated in the same way. The rate of increase of the total stress will be considered constant (linear variation of overburden), and the total stress in the dredgings layer is assumed constant during each time step. The corresponding stresses will be added at the end of each time step, and the results obtained from the solution of the field equations will be corrected, as previously explained.

SUMMARY

Developed in this chapter is a mathematical model that simulates the desiccation and consolidation of dredged material deposited continuously or periodically in a diked containment area. The flow of water

through a heterogeneous porous medium undergoing one-dimensional deformation, regardless of the degree of saturation, has been described by a nonlinear partial differential equation involving only assumptions that (a) the velocity of the solids can be neglected with respect to the velocity of the fluid and (b) the generalized form of Darcy's law holds throughout the process. The boundary conditions incorporated in the model include those natural conditions that are usually found in the field and certain artificial conditions that may be implemented to accelerate the dewatering process. However, due to a general lack of knowledge about the volume change characteristics of partially saturated soils (specifically, there is apparently no unique strain-stress relationship for such soils), this boundary value problem can not be solved directly; therefore, a step-by-step solution has been pursued. The assumptions involved in this technique are that, during a small time increment, Δt, the unsaturated zone of the dredgings layer is incompressible and the total stress on the dredgings layer does not change. Within this framework the boundary value problem can be reduced to a single variable nonlinear parabolic differential equation that can be solved numerically in the interval Δt. After a solution has been determined for a given time increment, a number of end-of-step corrections are made to incorporate into the results the fact that the assumptions mentioned above are not strictly correct. The method of solution described is essentially a numerical technique that will be developed in detail in Chapter 5.

CHAPTER 4

EXPERIMENTAL CHARACTERIZATION OF ENGINEERING PROPERTIES

Based on the results obtained from an extensive experimental program conducted during the past few years to describe the engineering properties of dredged materials, empirical relationships are developed for use in the associated computer program. Emphasis is given to dredgings from the area around Toledo, Ohio, because subsequent comparisons are made between settlements measured in an instrumented landfill near Toledo and those calculated by means of the mathematical model. The concept of scale heterogeneity is applied to account for possible changes in soil properties with depth.

PERMEABILITY

Throughout this work the term permeability has been used to describe two traditionally different, though physically similar, phenomena. The first, termed the saturated permeability, is normally used by soil engineers to characterize the flow of water through saturated soils, whereas the second, frequently called the unsaturated permeability or hydraulic conductivity, is usually employed by soil scientists to describe the flow of water in partially saturated soils. Although both of these properties characterize a common physical process, each is governed by different material considerations and thereby requires individualized study.

Saturated Permeability

While the concept of saturated permeability is very well known, the task of measuring its value with any appreciable degree of accuracy is extremely difficult. Different laboratory test procedures often yield widely differing results for the same soil, and the results obtained from

field tests frequentely bear no resemblence to those determined by laboratory tests. In this experimental program seven different methods were used to determine the saturated permeability of dredged materials; these include conventional consolidation, single load consolidation, slurry consolidation, direct permeability, gravity drainage, vacuum drainage, and field infiltration tests. Figure 4-1 summarizes the results obtained from all of these tests as a function of the void ratio, e, of the dredgings. The curve representing the average permeability can be approximated by

$$\log k = - \{(0.8\ e^2 - 10.2\ e + 62)^{\frac{1}{2}}\} \quad , \qquad (4\text{-}1)$$

where k is the coefficient of permeability in cm/sec and e is the void ratio. While it is true that some of the tests mentioned above are limited to different ranges of void ratio, most overlap somawhat in their ranges of applicability. In particular, the slurry consolidation test, which was performed on a large number of samples, yielded permeability values over a void ratio range from about 1 to 5. Certain simpler functions might have been chosen to describe the permeability over limited ranges of void ratio, but subsequent use of such expressions in the mathematical model indicated that the derivative of the permeability with respect to the void ratio was too large; consequently, Equation (4-1) was chosen to represent the saturated permeability - void ratio relationship in this problem.

Unsaturated Permeability

The permeability of any soil at a given void ratio is known to decrease rapidly as the degree of saturation decreases. Although no tests were performed in this experimental program to determine the relationship

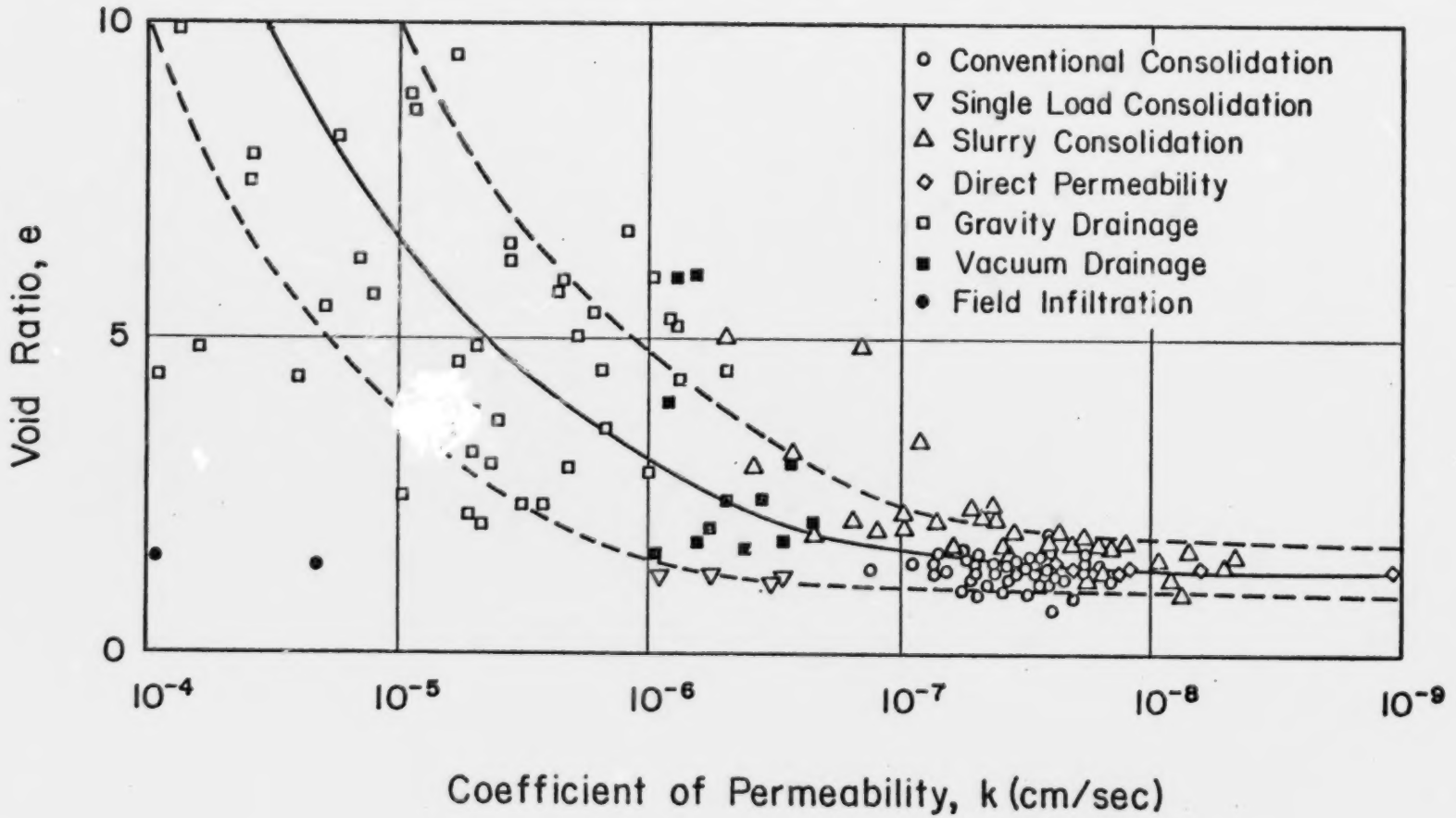

Figure 4-1. Summary of Values for Coefficient of Permeability

between the unsaturated permeability, k, and the degree of saturation, S, Scott (1963) reported that, for almost all types of soils, this function can be closely approximated over the entire range of saturation from 0 to 100% by

$$k = k_s S^{3.5} \quad , \tag{4-2}$$

where k_s is the saturated permeability at the same void ratio. Use of Equation (3-9) allows Equation (4-2) to be rewritten in terms of the volumetric water content, θ, as

$$k = k_s \left[\frac{1+e}{e} \theta \right]^{3.5} \quad . \tag{4-3}$$

Equation (4-3) approximates relatively well the general shape of the relationship proposed by Philip (1969) for the unsaturated permeability of fine-grained soils.

WATER RETENTION CHARACTERISTICS

One major factor affecting the behavior of partially saturated soils is their water retention characteristic, that is, the relationship between the water content and the soil-water potential (also called suction or capillary potential) at equilibrium. While this concept is quite common to soil scientists, it is relatively unfamiliar to soil engineers, who deal most frequently with saturated soils. Since water retention is a volumetric concept (Miller and Miller, 1956), the use of water content defined by weight (as usually done in soil engineering) is inadequate to define a water retention function. Unfortunately, the available data for the water retention characteristics of dredgings were expressed in

such a way, and only the initial and final void ratios were known. Figure 4-2 shows some of these relationships for dredgings from Toledo, Ohio (C22TO), Monroe, Michigan (C27MM), Detroit, Michigan (D1DM), and Cleveland, Ohio (D4CO); e_o and e_f represent the initial and final void ratios, respectively. Although it is not possible from the data given in Figure 4-2 to find a unique relationship between volumetric water content and soil-water potential, a maximum and a minimum functions can be found. If the volumetric water content, θ, is written as a function of the weight water content, w, by

$$\theta = \frac{G_s}{1+e} w \quad , \tag{4-4}$$

where G_s is the specific gravity of the solids, a minimum value of θ may be found for each value of the soil-water potential, ψ, by replacing e by e_o in Equation (4-4). Similarly, a maximum value of θ can be found by substituting e_f for e in Equation (4-4). These two limiting relationships are shown in Figure 4-3 for the four samples described previously; however, the exact water retention function is still not known. The function used in this study to represent the water retention characteristics of dredgings has been judiciously chosen as

$$\theta = \theta_s \exp\left[-\frac{\mu \psi}{\psi_{cr}}\right] + \theta_{cr} \tanh\left[\frac{\nu \psi}{\psi_{cr}}\right] \quad , \tag{4-5}$$

where θ is the volumetric water content corresponding to the soil-water potential, ψ, θ_s is the volumetric water content at saturation, θ_{cr} is the limiting or air-dry volumetric water content, ψ_{cr} is the limiting or air-dry soil-water potential, and μ and ν are two empirical parameters that depend on the particular material tested. For all dredged materials

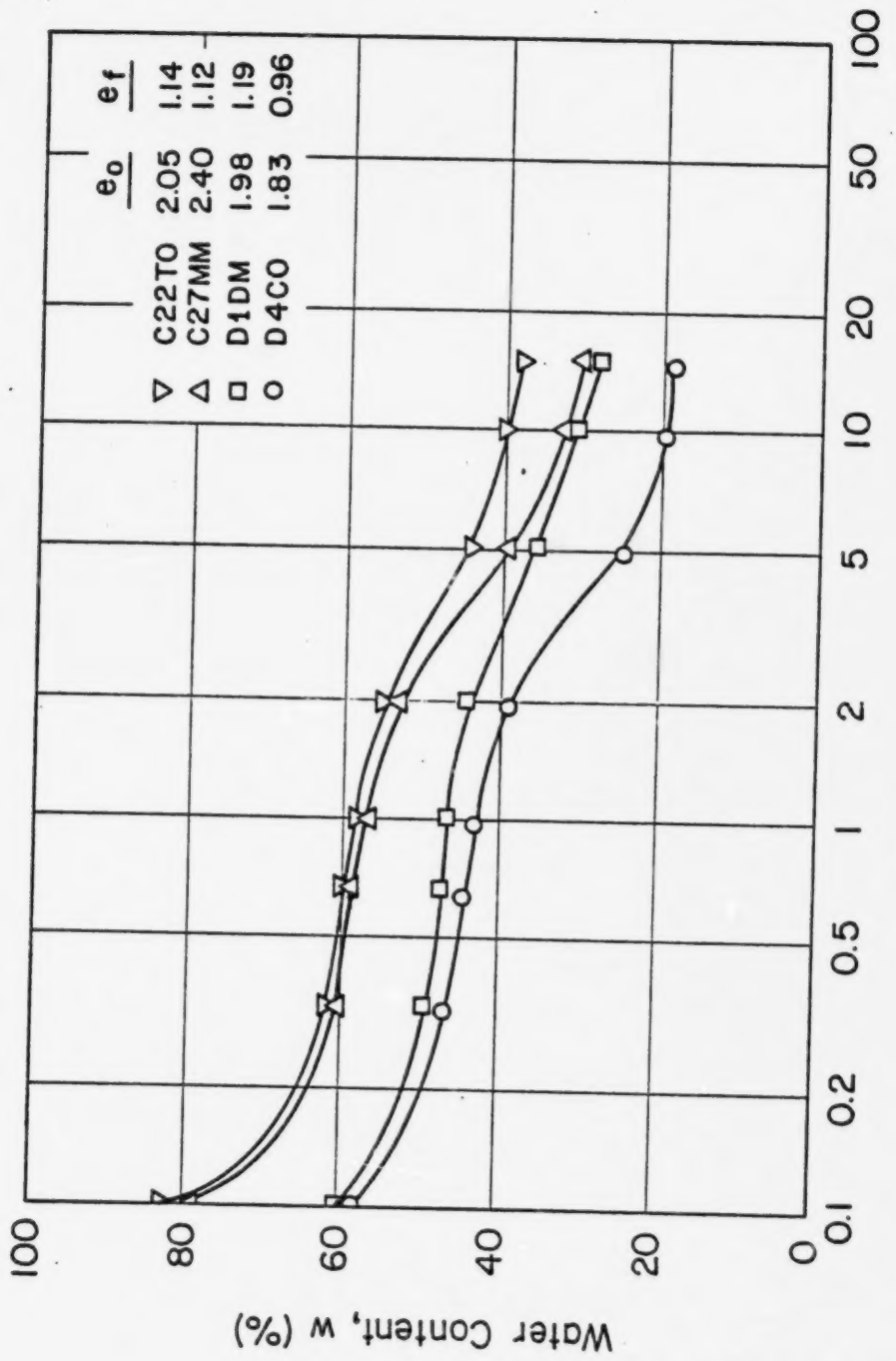

Figure 4-2. Characteristic Water Retention Curves

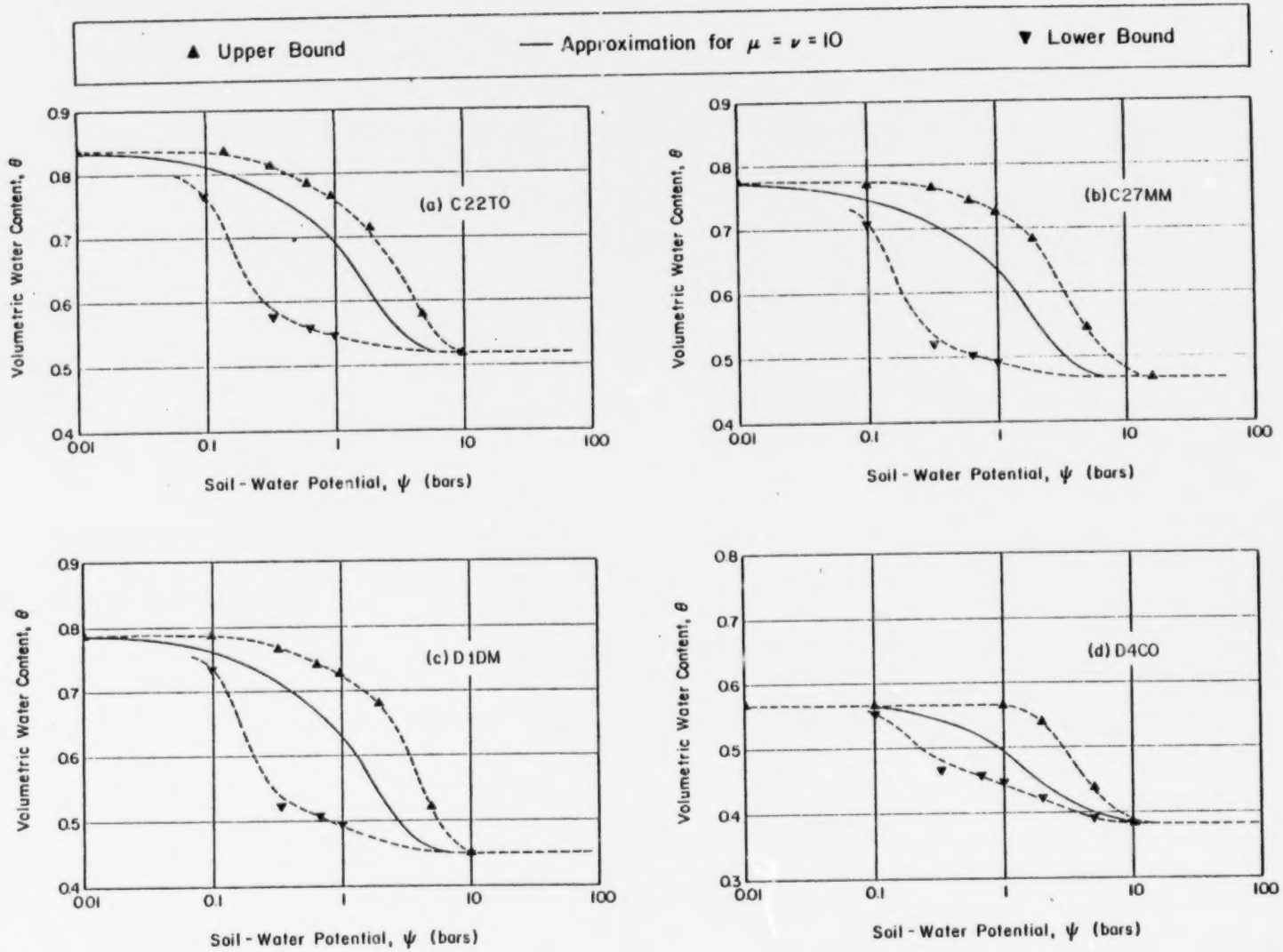

Figure 4-3. Volumetric Water Content versus Soil-Water Potential

analyzed herein, a value of 10 for both μ and ν gave satisfactory results, as illustrated in Figure 4-3. There were three principal reasons for choosing the type of functional relationship depicted in Equation (4-5). First, it complies with the specific water capacity requirement

$$C(\psi \rightarrow \infty) = \frac{d\theta}{d\psi}(\psi \rightarrow \infty) = 0 \quad , \tag{4-6}$$

where C is the specific water capacity; second, it is easier to program than the series of functions, erfc(ψ) and inverfc(θ), proposed by Philip (1960a, 1960b) and corresponding values are in reasonably good agreement; and third, it describes very well the relationships for at least a few other soils, as shown in Figure 4-4 for two Panoche clay loams (Nielsen, Davidson, Biggar, and Miller, 1964) and a Sarpy loam (Hanks and Bowers, 1962).

The specific water capacity function derived from Equation (4-5) is

$$C = -\frac{\mu\theta_s}{\psi_{cr}} \exp\left[-\frac{\mu\psi}{\psi_{cr}}\right] + \frac{\nu\theta_{cr}}{\psi_{cr}} \operatorname{sech}^2\left[\frac{\nu\psi}{\psi_{cr}}\right] \quad . \tag{4-7}$$

Note that ψ and ψ_{cr} must be introduced in Equation (4-7) as positive values. The limiting values θ_{cr} and ψ_{cr} in Equations (4-5) and (4-7) require some explanation. It is known that all water retention curves approach a minimum water content as the soil-water potential increases; this value of the water content, which is also known as the air-dry water content (Staple, 1971), is defined as θ_{cr} in Equations (4-5) and (4-7). The air-dry soil-water potential, ψ_{cr}, is that value of the soil-water potential (taken as an absolute value) at which the volumetric water content is essentially equal to the air-dry water content. It has been found

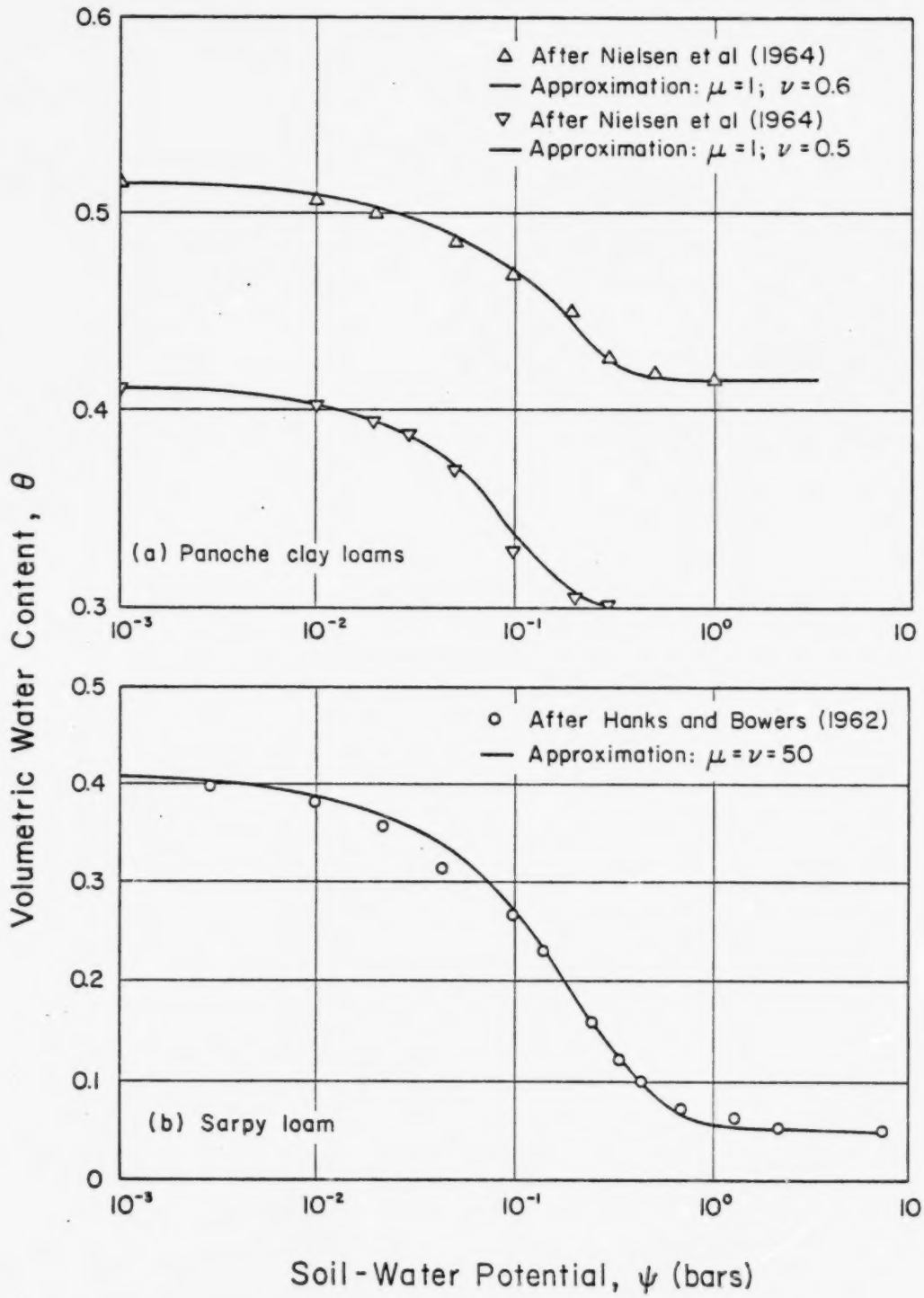

Figure 4-4. Water Retention Curves for Some Typical Soils

for most of the dredgings tested in this study that the relationship between θ_s and θ_{cr} is linear, as shown in Figure 4-5 (Sample C28CI was taken from Chicago, Illinois). This means that the air-dry water content of a given dredged material can be determined from a knowledge of its minimum void ratio at saturation (that is, the void ratio at the shrinkage limit), because, if the soil is fully saturated, its volumetric water content is equal to e/(1+e). Although the results obtained by use of Equations (4-5) and (4-7) in the mathematical model showed good agreement with the experimental data, as will be shown in Chapter 6, these equations should be regarded as tentative or preliminary in nature until more accurate data is gathered to better describe the water retention characteristics of dredged materials.

VOLUME CHANGE CHARACTERISTICS

As mentioned in Chapter 3, the physical phenomenon corresponding to the volume change associated with consolidation of a saturated soil is different from that which characterizes the volume change process that occurs in a partially saturated soil, and these two compressibilities will be discussed in the following sections.

Saturated Compressibility

The compressibility of saturated dredgings can be described by

$$e = e_o - C_c \log \frac{\bar{\sigma} + \Delta\bar{\sigma}}{\bar{\sigma}} \quad , \tag{4-8}$$

where e is the new void ratio, e_o is the initial void ratio, C_c is the compression index, and $\bar{\sigma}$ is the effective stress. Based on the results obtained from a large number of conventional consolidation tests that have been performed during this experimental program, the compression

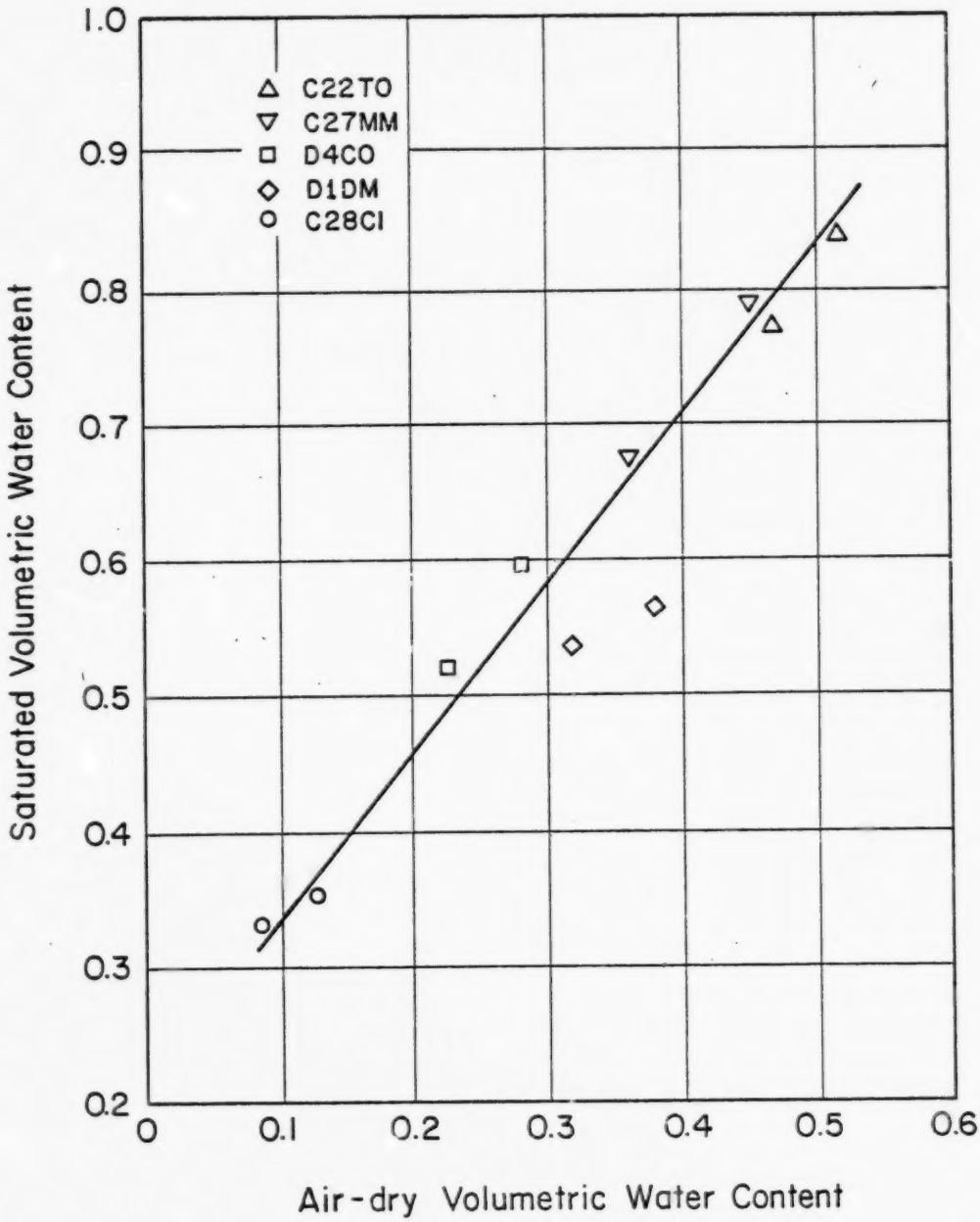

Figure 4-5. Relationship Between Saturated and Air-dry Water Contents

index, C_c, of dredged materials can be reasonably well represented by

$$C_c = 0.01(w - 7) \quad , \tag{4-9}$$

where w is the weight water content expressed as a percentage. If it is assumed that the specific gravity of the solids is 2.70, Equation (4-9) can be used to write the compression index as a function of the void ratio:

$$C_c = 0.01(37e - 7) \quad . \tag{4-10}$$

Since the compression index is a function of the void ratio, which is time-dependent, the derivative, $\partial C_c/\partial t$, will not be zero, thereby introducing a nonlinear factor in the governing differential equation. However, this problem is handled by means of a step-by-step, or incremental, approach. The spatial distribution of the compression index, $C_c(z)$, was assumed to be constant during any given time increment, and the value was adjusted after each time step by use of Equation (4-10) and the new void ratio distribution.

Unsaturated Compressibility

Since no data are available to describe the unsaturated compressibility of dredged materials, certain assumptions had to be made. The only volume change considered in the partially saturated zone is that due to the increase in total stress at the end of the step, as explained in Chapter 3; this means that any reduction in void ratio caused by an increase in the soil-water potential is neglected. This assumption is necessitated by (a) the apparent lack of information regarding the magnitude of Coleman's coefficients, C_1 and C_2, for dredged materials, or, in fact, for any soils, (b) the large amount of time and effort required to

obtain this type of data, and (c) the uncertainty associated with the use of such data, even if obtained. However, it is believed that the error introduced by this assumption is probably small because (a) the reduction in void ratio due to an increase in soil-water potential is small, except for very large values of this increase, as shown by the water retention data, and (b) the only part of the soil deposit subjected to large suction values is a relatively shallow zone at the evapotranspiration boundary. Future attention should be directed toward quantifying the volume change characteristics of partially saturated soils.

HETEROGENEITY

The heterogeneity of the deposit in the vertical direction has been handled by using the concept of scale heterogeneity. This concept implies that, if a layer of soil is of similar nature and origin (as is the case with the dredgings studied herein), any soil property, $\Gamma(z,t,x_1,x_2,\ldots)$, where z is the vertical coordinate, t is time, and x_1, x_2,... are general functions of z and t, can be expressed as

$$\Gamma(z,t,x_1,x_2,\ldots) = F_s(z)\Gamma^*(t,x_1,x_2,\ldots) \quad , \qquad (4\text{-}11)$$

where F_s is some particular function, and Γ^* is a general function that describes the same property as Γ at any point in the layer. This concept has been widely used by soil scientists (Miller and Miller, 1956; Klute and Wilkinson, 1958; Elrick, Scandrett, and Miller, 1959; Wilkinson and Klute, 1959; Philip, 1967; Watson and Whisler, 1972) and has two great advantages over any other method of heterogeneity description. First, it simplifies greatly the problem of data input and data handling in the computer program, and second, it allows F_s to be any type of function,

including discontinuous, probabilistic, and even an interpolating polynomial. Although heterogeneity has not been taken into consideration in the solutions described in Chapter 6 (due to the inability to characterize this heterogeneity in any conclusive manner), the program has the capability of handling scale heterogeneity functions for all soil properties.

SUMMARY

This chapter describes the results of an extensive experimental program aimed toward defining appropiate empirical relationships for the engineering properties of dredged materials for use in the mathematical formulation. Such relationships have been developed to approximate (a) saturated permeability as a function of void ratio, (b) unsaturated permeability as a function of void ratio and volumetric water content, (c) volumetric water content as a function of soil-water potential, (d) specific water capacity as a function of soil-water potential, and (e) compression index as a function of void ratio. All of these functions represent averages of data from a large number of tests.

The function describing the water retention characteristics of partially saturated dredgings was found to apply reasonably well to some other soils, thereby enhancing its validity to some extent. The decrease in void ratio due to an increase in the soil-water potential in the partially saturated zone is neglected in the mathematical model because there is insufficient information to realistically quantify the magnitude of such a volume reduction; however, the error introduced by this assumption is probably very small. The heterogeneity of the soil deposit has been handled by means of scale heterogeneity functions, one of which must be defined for each soil property. The time and expense required

to obtain all the data that the mathematical model can handle will generally be excessive, but it is believed that a number of reasonable assumptions, some of which are described in this chapter, can be made without causing large errors in the resulting calculations.

CHAPTER 5

NUMERICAL SOLUTION OF FLOW PROCESS

The nonlinear boundary value problem outlined in Chapter 3 is expressed in terms of dimensionless parameters and solved for generalized forms of the σ-t and e-ψ relationships by use of the numerical technique explained in this chapter. Also described are the stability and convergence characteristics of this particular numerical technique.

DIMENSIONLESS FORM OF THE PROBLEM

As defined by Equations (3-62) and (3-63), the vertical flow of water in a layer of dredged material is described by

$$\frac{0.4343\ C_c\ \gamma_w}{(1+e)(\bar{\sigma}_o+\sigma'-u)}\frac{\partial u}{\partial t}=\frac{\partial}{\partial z}\left[\ k\ \frac{\partial u}{\partial z}\ \right] \tag{5-1}$$

for the saturated zone and

$$C\ \frac{\partial \psi}{\partial t}=\frac{\partial}{\partial z}\left[\ k\ \frac{\partial \psi}{\partial z}+k\ \right] \tag{5-2}$$

for the partially saturated zone. In the subsequent manipulation of these equations it is convenient to define a dimensionless vertical coordinate, Z, and a dimensionless time parameter, T, as

$$Z=\frac{z}{H_\ell}\quad , \tag{5-3}$$

and

$$T=\frac{k_o\,tA}{H_1}\quad , \tag{5-4}$$

where H_1 is the thickness of the first layer, H_ℓ is the thickness of the ℓ^{th} layer, k_o is the initial permeability of the saturated material at the bottom (z=0) of the first layer, and A is a dimensionless coefficient

termed the "reduced time factor". The introduction of A in the definition of T serves two purposes; first, it incorporates in the formulation an inherent stability criterion, as will be explained later, and second, it allows the increment of real time, Δt, to be regulated without modifying the dimensionless time increment, ΔT. For example, if $k_o = 10^{-7}$ cm/sec and $H_1 = 300$ cm, a choice of $\Delta T = 0.01$ gives a real time increment, Δt, of

$$\Delta t = \frac{H_1 \Delta T}{A k_o} = \frac{0.01(300)}{10^{-7} A} = \frac{3 \times 10^7}{A} \text{ seconds} \quad , \tag{5-5}$$

which, without the introduction of the reduced time factor (that is, A = 1), yields a real time increment of about 8300 hours. However, the selection of A = 800 decreases the real time increment to about 10 hours, which is preferable for a problem of the type considered herein. Use of Equations (5-3) and (5-4) allows the time and space derivatives to be expressed as

$$\frac{\partial}{\partial t} = \frac{A k_o}{H_1} \frac{\partial}{\partial T} \tag{5-6}$$

and

$$\frac{\partial}{\partial z} = \frac{1}{H_\ell} \frac{\partial}{\partial Z} \quad . \tag{5-7}$$

If the dimensionless water head, h, is defined as

$$h = \frac{\psi}{H_1} \tag{5-8}$$

in the partially saturated zone and

$$h = \frac{u}{\gamma_w H_1} \tag{5-9}$$

in the saturated zone, and if the dimensionless permeability, k^*, is defined as k/k_o, Equations (5-1) and (5-2) can be written as

$$\frac{0.4343\ C_c \gamma_w}{(1+e)(\bar{\sigma}_o + \sigma - h\gamma_w H_1)} \frac{Ak_o}{H_1} \frac{\partial(h\gamma_w H_1)}{\partial T} = \frac{1}{H_\ell} \frac{\partial}{\partial Z} \left[k^* k_o \frac{1}{H_\ell} \frac{\partial(h\gamma_w H_1)}{\partial Z} \right] \quad , \tag{5-10}$$

and

$$C \frac{Ak_o}{H_1} \frac{\partial(hH_1)}{\partial T} = \frac{1}{H_\ell} \frac{\partial}{\partial Z} \left[k^* k_o \frac{1}{H_\ell} \frac{\partial(hH_1)}{\partial Z} + k^* k_o \right] \quad . \tag{5-11}$$

With the definition of the dimensionless initial effective pressure, $\bar{p}_o = \bar{\sigma}_o / \gamma_w H_1$, the dimensionless total pressure, $p = \sigma / \gamma_w H_1$, and the dimensionless thickness ratio, $R_\ell = H_1 / H_\ell$, Equation (5-10) which describes the flow of water through the saturated zone, can be further reduced to

$$\frac{0.4343\ C_c A}{(1+e)} \frac{\partial h}{\partial T} = R_\ell^2 (\bar{p}_o + p - h) \frac{\partial}{\partial Z} \left(k^* \frac{\partial h}{\partial Z} \right) \quad . \tag{5-12}$$

The definition of the dimensionless specific water capacity, $C^* = CH_1$, allows Equation (5-11), which describes the flow of water through the partially saturated zone, to be simplified to

$$C^* A \frac{\partial h}{\partial T} = R_\ell \frac{\partial}{\partial Z} \left(R_\ell k^* \frac{\partial h}{\partial Z} + k^* \right) \quad . \tag{5-13}$$

The introduction of Equations (5-6), (5-7), (5-8), and (5-9) into the boundary conditions yields the following dimensionless expressions.

Upper Boundary

Evapotranspiration condition:

$$k^* \left(R_\ell \frac{\partial h}{\partial Z} + 1 \right) = - \frac{W_w}{k_o} \quad \text{for} \quad T \geq 0 \quad . \tag{5-14}$$

Infiltration condition:

$$h = 0 \quad \text{for} \quad T \geq 0 \quad . \tag{5-15}$$

Inactive boundary condition:

$$h = \frac{\Psi_{cr}}{H_1} \quad \text{for} \quad W_w > 0 \quad \text{and} \quad T \geq 0 \quad . \tag{5-16}$$

Lower Boundary

Free drainage:

$$h = 0 \quad \text{for } Z = 0 \text{ and } T \geq 0 \quad . \tag{5-17}$$

Impervious surface:

$$\frac{\partial h}{\partial Z} = 0 \quad \text{for} \quad Z = 0 \text{ and } T \geq 0 \quad . \tag{5-18}$$

Impeded drainage:

$$R_{\ell} \frac{\partial h}{\partial Z} - \lambda H_1 h = 0 \quad \text{for } Z = 0 \text{ and } T \geq 0 \quad . \tag{5-19}$$

Forced drainage:

$$h = \frac{-h_v}{\gamma_w H_1} \quad . \tag{5-20}$$

Interface Between Two Layers

Saturated soil:

$$\left(Rk^* \frac{\partial h}{\partial Z}\right)_{\ell-1} = \left(Rk^* \frac{\partial h}{\partial Z}\right)_{\ell} \quad . \tag{5-21}$$

Partially saturated soil:

$$\left[k^*\left(R \frac{\partial h}{\partial Z} + 1\right)\right]_{\ell-1} = \left[k^*\left(R \frac{\partial h}{\partial Z} + 1\right)\right]_{\ell} \tag{5-22}$$

where ℓ and ℓ-1 represent the ℓ^{th} and $(\ell-1)^{th}$ layer, respectively. As a consequence of the dimensionless variables introduced by Equations (5-6), (5-7), (5-8), and (5-9), each layer extends from Z = 0 to Z = 1, regardless of the actual thickness of the layer, and the water pressures in each layer can be compared directly, because their dimensionless values are independent of layer thickness.

DISCRETIZATION TECHNIQUE

In order to solve Equations (5-12) and (5-13) subject to certain of the conditions represented by Equations (5-14) through (5-20), a finite difference approach has been used and the parabolic equations have been discretized according to the well known Crank-Nicolson scheme (Crank and Nicolson, 1947), which is an implicit technique consisting of the discretization of the function and its derivatives at a point $(i, j + \frac{1}{2})$ that is not in the finite difference grid (in this case i and j represent space and time, respectively), as shown by Figure 5-1. Hence, the value of the function, h, its time derivative, $\partial h/\partial T$, and its space derivatives, $\partial h/\partial Z$ and $\partial^2 h/\partial Z^2$, at point A can be approximated by

$$h_{i,j+\frac{1}{2}} = \frac{1}{2}\left(h_{i,j} + h_{i,j+1}\right) \tag{5-23}$$

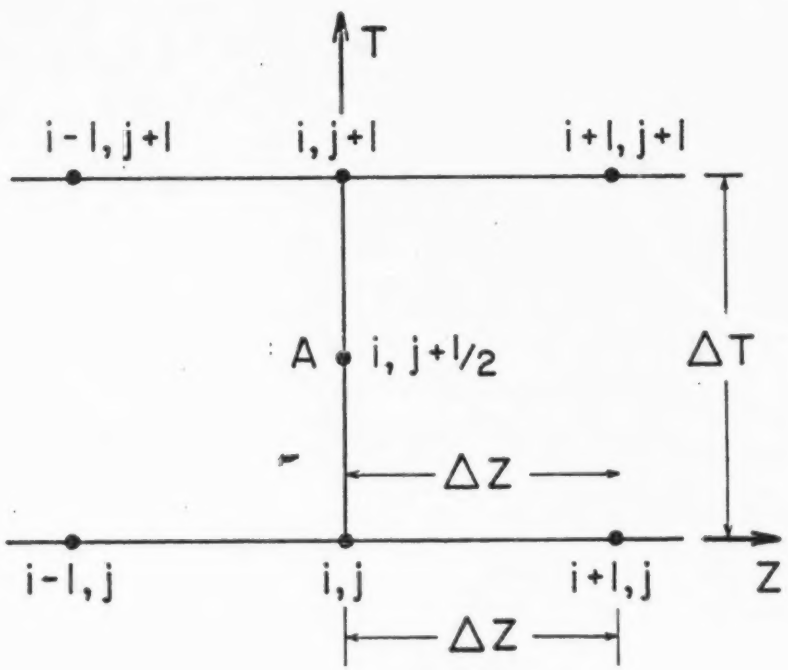

Figure 5-1. Crank-Nicolson Discretization Scheme

$$\left.\frac{\partial h}{\partial T}\right|_{i,j+\frac{1}{2}} = \frac{h_{i,j+1} - h_{i,j}}{\Delta T} \quad , \tag{5-24}$$

$$\left.\frac{\partial h}{\partial Z}\right|_{i,j+\frac{1}{2}} = \frac{1}{2}\left[\frac{h_{i+1,j+1} - h_{i-1,j+1}}{2\Delta Z} + \frac{h_{i+1,j} - h_{i-1,j}}{2\Delta Z}\right] \tag{5-25}$$

and

$$\left.\frac{\partial^2 h}{\partial Z^2}\right|_{i,j+\frac{1}{2}} = \frac{1}{2}\left[\frac{h_{i+1,j+1} - 2h_{i,j+1} + h_{i-1,j+1}}{\Delta Z^2} + \frac{h_{i+1,j} - 2h_{i,j} + h_{i-1,j}}{\Delta Z^2}\right]. \tag{5-26}$$

Due to the expansion of the right-hand side of Equation (5-12), two nonlinear terms appear, namely, $-k^*h(\partial^2 h/\partial Z^2)$ and $-h(\partial k^*/\partial Z)(\partial h/\partial Z)$, and the discretization of these two terms according to the scheme mentioned above would result in

$$-k^*\frac{1}{4}(h_{i,j} + h_{i,j+1})\left[\frac{h_{i+1,j+1} - 2h_{i,j+1} + h_{i-1,j+1}}{\Delta Z^2} + \frac{h_{i+1,j} - 2h_{i,j} + h_{i-1,j}}{\Delta Z^2}\right]$$

and

$$\frac{\partial k^*}{\partial Z}\frac{1}{4}(h_{i,j} + h_{i,j+1})\left[\frac{h_{i+1,j+1} - h_{i-1,j+1}}{2\Delta Z} + \frac{h_{i+1,j} - h_{i-1,j}}{2\Delta Z}\right] \quad ,$$

which will lead to a system of algebraic equations with nonlinear terms in $h_{i,j+1}h_{i+1,j+1}$, $h^2_{i,j+1}$, and $h_{i,j+1}h_{i-1,j+1}$. Although this system of equations could be solved by an iterative technique (such as the Newton-Raphson method), the nonlinearity presents two problems; first, no stability or convergence criteria can generally be establish for the differential equation, and second, the particular iterative technique selected will depend on the coefficients characterizing the system of equations, thereby making it difficult to develop a program that is sufficiently general to account for the different types of dredged materials. The same reasoning can be extended to the nonlinear term of Equation (5-13), namely, $\partial/\partial Z\{k^*(Z,h)\,\partial h/\partial Z\}$, which, after expansion, can be written as

$k^*\partial^2 h/\partial Z^2 + \partial k^*/\partial Z\ \partial h/\partial Z + \partial k^*/\partial h(\partial h/\partial Z)^2$. As a result of the above mentioned drawbacks, the equations were linearized by assuming that one of the factors in the nonlinear term is constant, solving the associated linear system of equations, and, once a solution has been found, iterating this solution. For example, the discretization of the nonlinear term $-k^*h(\partial^2 h/\partial Z^2)$ of Equation (5-12) can be assumed to be

$$-k^*_i{}^{(o)}\ \frac{1}{2}\left[\frac{h_{i+1,j+1}-2h_{i,j+1}+h_{i-1,j+1}}{\Delta Z^2}+\frac{h_{i+1,j}-2h_{i,j}+h_{i-1,j}}{\Delta Z^2}\right] ,$$

where $h_i^{(o)}$ is some constant value which, in this case, can be taken equal to $h_{i,j}$; then, the solution of the linear system of equations will be

$$h = h^{(1)}_{i,j+1} \ ; \qquad i = 1,n \ . \tag{5-27}$$

By assuming now a new value of

$$h_i^{(1)} = \frac{h_{i,j}+h^{(1)}_{i,j+1}}{2} \tag{5-28}$$

for the linearized term, we may solve the system and get another solution

$$h = h^{(2)}_{i,j+1} \ ; \qquad i = 1,n \ . \tag{5-29}$$

This iteration procedure is repeated until convergence is obtained, namely

$$\left|h_i^{(r)} - \frac{h_{i,j}+h^{(r)}_{i,j+1}}{2}\right| \le \epsilon \ , \tag{5-30}$$

where ϵ is some arbitrarily small number. This technique will allow (a) the establishment of some type of stability and convergence criteria for the numerical solution of the differential equations, and (b) the solution of the system of equations by means of a completely general elimination algorithm. By use of this linearization technique, Equations (5-12) and (5-13) may be rewritten as

$$\frac{0.4343\ C_c A}{(1+e)}\frac{\partial h}{\partial T} = R_\ell^2\ (\bar{p}_o + p - h^*)\left[\ k^*(Z)\ \frac{\partial^2 h}{\partial Z^2} + \frac{\partial k^*(Z)}{\partial Z}\ \frac{\partial h}{\partial Z}\ \right] \tag{5-31}$$

and

$$C^*_A \frac{\partial h}{\partial T} = R_\ell^2 k^*(Z,h^*) \frac{\partial^2 h}{\partial Z^2} + R_\ell \frac{\partial k^*(Z,h^*)}{\partial Z} \left[R_\ell \frac{\partial h}{\partial Z} + 1 \right] , \quad (5\text{-}32)$$

where h^* represents the linearized water head, $h^{(r)}_{i,j}$. The introduction of the Crank-Nicolson scheme, represented by Equations (5-23) to (5-26), into the above equations yields

$$\frac{0.4343\ C_{c(i,j)}A}{(1+e)} \left[\frac{h_{i,j+1} - h_{i,j}}{\Delta T} \right] = R_\ell^2 (\bar{p}_o + p - h^*) \left\{ \frac{k^*_{i,j}}{2} \left[\frac{h_{i+1,j} - 2h_{i,j} + h_{i-1,j}}{\Delta Z^2} + \frac{h_{i+1,j+1} - 2h_{i,j+1} + h_{i-1,j+1}}{\Delta Z^2} \right] + \frac{1}{2} \frac{\partial k^*_{i,j}}{\partial Z} \left[\frac{h_{i+1,j+1} - h_{i-1,j+1}}{2\Delta Z} + \frac{h_{i+1,j} - h_{i-1,j}}{2\Delta Z} \right] \right\} \quad (5\text{-}33)$$

and

$$C^*_{i,j}A \left[\frac{h_{i,j+1} - h_{i,j}}{\Delta T} \right] = R_\ell^2 \frac{k^*_{i,j}}{2} \left[\frac{h_{i+1,j+1} - 2h_{i,j+1} + h_{i-1,j+1}}{\Delta Z^2} + \frac{h_{i+1,j} - 2h_{i,j} + h_{i-1,j}}{\Delta Z^2} \right] + \frac{R_\ell^2}{2} \frac{\partial k^*_{i,j}}{\partial Z} \left[\frac{h_{i+1,j+1} - h_{i-1,j+1}}{2\Delta Z} + \frac{h_{i+1,j} - h_{i-1,j}}{2\Delta Z} \right] + R_\ell \frac{\partial k^*_{i,j}}{\partial Z} . \quad (5\text{-}34)$$

If each integration domain (that is, each dredging layer) is divided into n intervals, Equations (5-33) and (5-34) represent a system of N(n-1) algebraic equations (where N is the number of integrations domains) with Nn + 1 unknowns. This is the case because Equations (5-33) and (5-34) can not be directly applied to the points on the boundaries, nor can they be applied to the points at the interface between two dredgings layers. The other N + 1 equations will be obtained at these particular points

Boundary Conditions

The boundary conditions in a parabolic boundary value problem are usually one of the following types:

Dirichlet Condition $h = d$, (5-35a)

Neumann Condition $ah_n + bh_s = d$, (5-35b)

Mixed Condition $ah_n + bh_s + ch = d$, (5-35c)

where h is the function, h_n and h_s are its normal and tangential derivatives, and a, b, c, and d are constants or variable coefficients. Accordingly, the boundary conditions described by Equations (5-15), (5-16), (5-17), and (5-20) are of the Dirichlet type; the ones represented by Equations (5-14) and (5-18) are of the Neumann type; and Equation (5-19) is a mixed boundary condition. The introduction of the Dirichlet conditions does not present any problem whatsoever, but the Neumann and mixed boundary conditions require some elaboration. Suppose that a mixed boundary condition, as described by Equation (5-35c), holds on the lower boundary, as shown in Figure 5-2, for the case where Equation (5-12) is the governing field equation. Since b = 0 for all cases, Equation (5-35c) can be reduced to

$$a \left.\frac{\partial h}{\partial Z}\right|_{1, j+\frac{1}{2}} + c\, h_{1, j+\frac{1}{2}} = d \quad ; \qquad t \geq 0 \quad . \tag{5-36}$$

Expanding $h_{2, j+\frac{1}{2}}$ in a Taylor series, we get

$$h_{2, j+\frac{1}{2}} = h_{1, j+\frac{1}{2}} + \Delta Z \left.\frac{\partial h}{\partial Z}\right|_{1, j+\frac{1}{2}} + \frac{\Delta Z^2}{2} \left.\frac{\partial^2 h}{\partial Z^2}\right|_{1, j+\frac{1}{2}} + O\{(\Delta Z)^3\} \ , \tag{5-37}$$

where $O\{(\Delta Z)^3\}$ represents the error introduced when higher order terms are neglected. Substitution of Equation (5-36) into Equation (5-37) gives

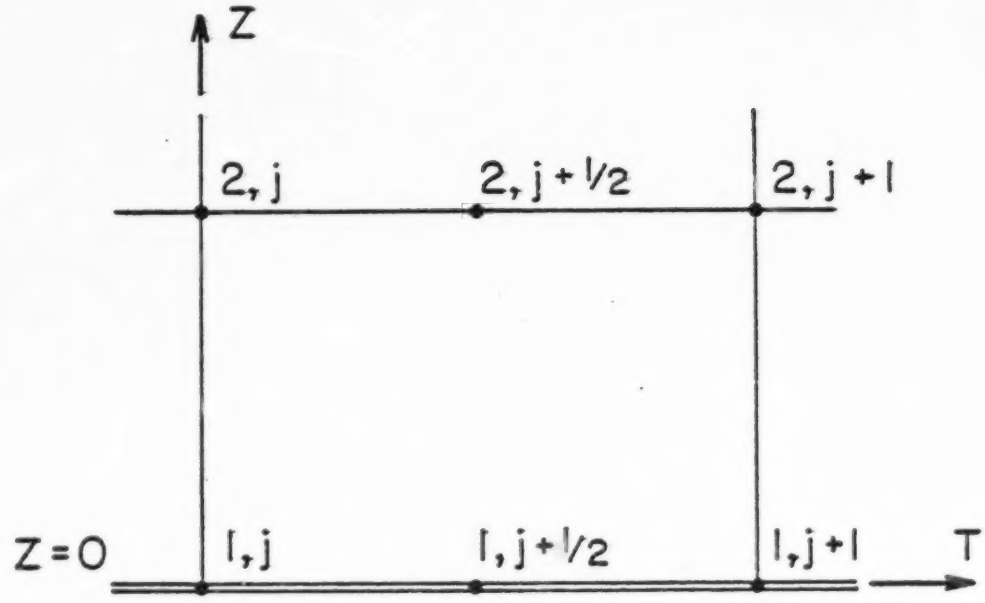

Figure 5-2. Boundary Points

$$\left.\frac{\partial^2 h}{\partial Z^2}\right|_{1,j+\frac{1}{2}} = \frac{2}{\Delta Z^2}\left[h_{2,j+\frac{1}{2}} - h_{1,j+\frac{1}{2}} - \frac{\Delta Z}{a}(d - c\, h_{1,j+\frac{1}{2}})\right] \quad , \tag{5-38}$$

and use of this expression for the second derivative in Equation (5-12) yields

$$\frac{0.4343\ C_c A}{(1+e)}\frac{\partial h}{\partial T} = R_{\ell}^2(\bar{p}_o + p - h^*)\left\{\frac{2k^*}{\Delta Z^2}\left[h_{2,j+\frac{1}{2}} - h_{1,j+\frac{1}{2}} - \frac{\Delta Z}{a}(d - c\, h_{1,j+\frac{1}{2}})\right] + \right.$$

$$\left.\frac{\partial k^*}{\partial Z}\left.\frac{\partial h}{\partial Z}\right|_{1,j+\frac{1}{2}}\right\} \quad , \tag{5-39}$$

which, by expressing the time derivative in finite difference form and by introducing again the boundary condition described by Equation (5-36), can be written as

$$\frac{0.4343\ C_c A}{(1+e)}\left(\frac{h_{1,j+1}-h_{1,j}}{\Delta T}\right)=R_\ell^2(\bar{p}_o+p-h^*)\left\{\frac{2k^*}{\Delta Z^2}\left[h_{2,j+\frac{1}{2}}-h_{1,j+\frac{1}{2}}-\right.\right.$$

$$\left.\left.\frac{\Delta Z}{a}(d-c\,h_{1,j+\frac{1}{2}})\right]+\frac{\partial k^*}{\partial Z}\left(\frac{d}{a}-\frac{c}{a}\,h_{1,j+\frac{1}{2}}\right)\right\}\quad . \tag{5-40}$$

If the soil is partially saturated at the boundary, the combination of Equations (5-13) and (5-38) yields

$$C^* A\,\frac{\partial h}{\partial T}=R_\ell^2 k^*\left\{\frac{2}{\Delta Z^2}\left[h_{2,j+\frac{1}{2}}-h_{1,j+\frac{1}{2}}-\frac{\Delta Z}{a}(d-c\,h_{1,j+\frac{1}{2}})\right]\right\}+$$

$$R_\ell^2\,\frac{\partial k^*}{\partial Z}\,\frac{\partial h}{\partial Z}\Big|_{1,j+\frac{1}{2}}+R_\ell\,\frac{\partial k^*}{\partial Z}\quad , \tag{5-41}$$

which, by applying the discretization scheme and introducing the boundary condition given by Equation (5-36), can be written as

$$C^* A\left(\frac{h_{1,j+1}-h_{1,j}}{\Delta T}\right)=R_\ell^2 k^*\left\{\frac{2}{\Delta Z^2}\left[h_{2,j+\frac{1}{2}}-h_{1,j+\frac{1}{2}}-\frac{\Delta Z}{a}(d-c\,h_{1,j+\frac{1}{2}})\right]\right\}+$$

$$R_\ell^2\,\frac{\partial k^*}{\partial Z}\left(\frac{d}{a}-\frac{c}{a}\,h_{1,j+\frac{1}{2}}\right)+R_\ell\,\frac{\partial k^*}{\partial Z}\quad . \tag{5-42}$$

Use of this same procedure allows one to write the equations at the upper boundary point $(m, j+\frac{1}{2})$ as

$$\frac{0.4343\ C_c A}{(1+e)}\left(\frac{h_{m,j+1}-h_{m,j}}{\Delta T}\right)=R_\ell^2(\bar{p}_o+p-h^*)\left\{\frac{2k^*}{\Delta Z^2}\left[h_{m-1,j+\frac{1}{2}}-h_{m,j+\frac{1}{2}}+\right.\right.$$

$$\left.\left.\frac{\Delta Z}{a}(d-c\,h_{m,j+\frac{1}{2}})\right]+\frac{\partial k^*}{\partial Z}\left(\frac{d}{a}-\frac{c}{a}\,h_{m,j+\frac{1}{2}}\right)\right\} \tag{5-43}$$

for the saturated soil and

$$C^* A\left(\frac{h_{m,j+1}-h_{m,j}}{\Delta T}\right)=R_\ell^2 k^*\left\{\frac{2}{\Delta Z^2}\left[h_{m-1,j+\frac{1}{2}}-h_{m,j+\frac{1}{2}}+\frac{\Delta Z}{a}(d-c\,h_{m,j+\frac{1}{2}})\right]\right\}+$$

$$R_\ell^2\,\frac{\partial k^*}{\partial Z}\left(\frac{d}{a}-\frac{c}{a}\,h_{m,j+\frac{1}{2}}\right)+R_\ell\,\frac{\partial k^*}{\partial Z} \tag{5-44}$$

for the partially saturated soil. This type of approximation for the boundary conditions affords four advantages over the normally used second order difference scheme, which in the case of the lower boundary will be

$$\frac{a}{2}\left(\frac{h_{2,j+1} - h_{o,j+1}}{2\Delta Z} + \frac{h_{2,j} - h_{o,j}}{2\Delta Z}\right) + c\left(\frac{h_{1,j+1} + h_{1,j}}{2}\right) = d \quad . \tag{5-45}$$

First, it combines the boundary condition with the discretized governing equation at the boundary, whereas the second order scheme is independent of the physical process (Equation (5-45) describes a mixed boundary condition, regardless of the physical process that takes place in the integration domain). Second, it is not necessary to introduce a point (o,j) outside the integration domain; in addition to increasing the amount of computer time and storage needed, this artifice represents a purely numerical technique without any physical meaning. Third, the error involved in the Taylor expansion, $O[(\Delta Z)^3]$, is one order of magnitude smaller than that produced by the discretization of the differential equation, $O[(\Delta Z)^2 + (\Delta T)^2]$; this means that no significant error will be introduced by the numerical approximation to the boundary condition. And fourth, Equations (5-40), (5-42), (5-43), and (5-45) are written as functions of the point on the boundary and the nearest grid point, thereby combining with the interior difference equations to produce a banded matrix of coefficients (band width equal to 3); therefore, a simple Gaussian elimination technique can be used to solve the system of equations, thus reducing the truncation errors inherent to all iterative methods and decreasing the time of operation. The use of the two boundary conditions reduces the number of unsatisfied unknowns to N-1, which is the number of points at the interfaces between the N different layers.

Interface Between Layers

In addition to the uniqueness of pressures at the interface between two different layers, the continuity of flow can be expressed by Equation (5-21) or (5-22), depending upon the saturation characteristics. Suppose that the interface is situated at point (i,j) shown in Figure 5-3. From the Taylor expansion around $(i,j+\frac{1}{2})$, we get approximately

$$h_{i-1,j+\frac{1}{2}} = h_{i,j+\frac{1}{2}} - \Delta Z \left.\frac{\partial h}{\partial Z}\right|^{\ell-1}_{i,j+\frac{1}{2}} + \frac{\Delta Z^2}{2} \left.\frac{\partial^2 h}{\partial Z^2}\right|^{\ell-1}_{i,j+\frac{1}{2}} \qquad (5\text{-}46a)$$

and

$$h_{i+1,j+\frac{1}{2}} = h_{i,j+\frac{1}{2}} + \Delta Z \left.\frac{\partial h}{\partial Z}\right|^{\ell}_{i,j+\frac{1}{2}} + \frac{\Delta Z^2}{2} \left.\frac{\partial^2 h}{\partial Z^2}\right|^{\ell}_{i,j+\frac{1}{2}} , \qquad (5\text{-}46b)$$

where ℓ and ℓ-1 represent the ℓ^{th} and $(\ell-1)^{th}$ layer, respectively. The second derivative at the interface

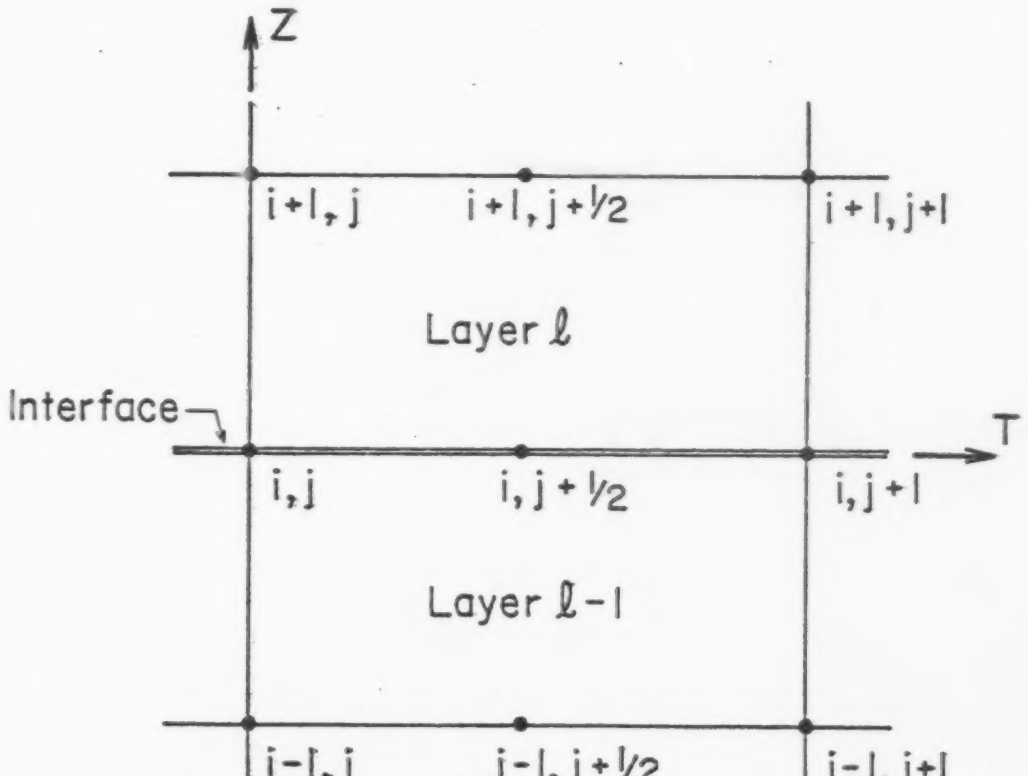

Figure 5-3. Grid Points at Interface Between Two Layers

may then be expressed as

$$\left.\frac{\partial^2 h}{\partial Z^2}\right|_{i,j+\frac{1}{2}}^{\ell-1} = \frac{2}{\Delta Z^2}\left[h_{i-1,j+\frac{1}{2}} - h_{i,j+\frac{1}{2}} + \Delta Z \left.\frac{\partial h}{\partial Z}\right|_{i,j+\frac{1}{2}}^{\ell-1} \right] \qquad (5\text{-}47a)$$

for the $(\ell-1)^{th}$ layer and

$$\left.\frac{\partial^2 h}{\partial Z^2}\right|_{i,j+\frac{1}{2}}^{\ell} = \frac{2}{\Delta Z^2}\left[h_{i+1,j+\frac{1}{2}} - h_{i,j+\frac{1}{2}} - \Delta Z \left.\frac{\partial h}{\partial Z}\right|_{i,j+\frac{1}{2}}^{\ell} \right] \qquad (5\text{-}47b)$$

for the ℓ^{th} layer. If the material is saturated at point (i,j), Equation (5-12) will hold for both layers, but different coefficients will apply for each layer. The introduction of Equations (5-47a) and (5-47b) into Equation (5-12) yields

$$\frac{0.4343\ C_c^{\ell-1}A}{(1+e_{\ell-1})}\ \frac{\partial h}{\partial T} = R_{\ell-1}^2(\bar{p}_o + p - h^*)\left\{k^*_{\ell-1}\left[\frac{2}{\Delta Z^2}\left(h_{i-1,j+\frac{1}{2}} - h_{i,j+\frac{1}{2}} + \Delta Z \left.\frac{\partial h}{\partial Z}\right|_{i,j+\frac{1}{2}}^{\ell-1}\right)\right] + \frac{\partial k^*_{\ell-1}}{\partial Z}\left.\frac{\partial h}{\partial Z}\right|_{i,j+\frac{1}{2}}^{\ell-1}\right\} \qquad (5\text{-}48a)$$

and

$$\frac{0.4343\ C_c^{\ell}A}{(1+e_{\ell})}\ \frac{\partial h}{\partial T} = R_{\ell}^2(\bar{p}_o + p - h^*)\left\{k^*_{\ell}\left[\frac{2}{\Delta Z^2}\left(h_{i+1,j+\frac{1}{2}} - h_{i,j+\frac{1}{2}} - \Delta Z \left.\frac{\partial h}{\partial Z}\right|_{i,j+\frac{1}{2}}^{\ell}\right)\right] + \frac{\partial k^*_{\ell}}{\partial Z}\left.\frac{\partial h}{\partial Z}\right|_{i,j+\frac{1}{2}}^{\ell}\right\} . \qquad (5\text{-}48b)$$

From Equation (5-21) we obtain

$$\left.\frac{\partial h}{\partial Z}\right|_{i,j+\frac{1}{2}}^{\ell-1} = \frac{R_\ell k^*_\ell}{R_{\ell-1} k^*_{\ell-1}}\left.\frac{\partial h}{\partial Z}\right|_{i,j+\frac{1}{2}}^{\ell} , \qquad (5\text{-}49)$$

which, together with Equation (5-48a) gives

$$\frac{0.4343\ C_c^{\ell-1}A}{(1+e_{\ell-1})}\ \frac{\partial h}{\partial T} = R_{\ell-1}^2(\bar{p}_o + p - h^*)\left\{k^*_{\ell-1}\left[\frac{2}{\Delta Z^2}\left(h_{i-1,j+\frac{1}{2}} - h_{i,j+\frac{1}{2}} + \Delta Z \frac{R_\ell k^*_\ell}{R_{\ell-1} k^*_{\ell-1}}\left.\frac{\partial h}{\partial Z}\right|_{i,j+\frac{1}{2}}^{\ell}\right)\right] + \frac{\partial k^*_{\ell-1}}{\partial Z}\frac{R_\ell k^*_\ell}{R_{\ell-1} k^*_{\ell-1}}\left.\frac{\partial h}{\partial Z}\right|_{i,j+\frac{1}{2}}^{\ell}\right\} . \qquad (5\text{-}50)$$

Equations (5-50) and (5-48b) may be rewritten as

$$\frac{0.4343\ C_c^{\ell-1}A}{(1+e_{\ell-1})}\ \frac{\partial h}{\partial T} - R_{\ell-1}^2(\overline{p}_o + p - h^*)\ \frac{2k^*_{\ell-1}}{\Delta Z^2}\ (h_{i-1,j+\frac{1}{2}} - h_{i,j+\frac{1}{2}}) =$$

$$R_{\ell-1}^2(\overline{p}_o + p - h^*)\ \frac{R_\ell k^*_\ell}{R_{\ell-1}k^*_{\ell-1}}\left[\ \frac{2k^*_{\ell-1}}{\Delta Z} + \frac{\partial k^*_{\ell-1}}{\partial Z}\ \right]\ \frac{\partial h}{\partial Z}\Big|^{\ell}_{i,j+\frac{1}{2}} \qquad (5\text{-}51a)$$

and

$$\frac{0.4343\ C_c^{\ell}A}{(1+e_{\ell})}\ \frac{\partial h}{\partial T} - R_{\ell}^2(\overline{p}_o + p - h^*)\ \frac{2k^*_{\ell}}{\Delta Z^2}\ (h_{i+1,j+\frac{1}{2}} - h_{i,j+\frac{1}{2}}) =$$

$$R_{\ell}^2\ (\overline{p}_o + p - h^*)\left[\ -\frac{2k^*_{\ell}}{\Delta Z} + \frac{\partial k^*_{\ell}}{\partial Z}\ \right]\ \frac{\partial h}{\partial Z}\Big|^{\ell}_{i,j+\frac{1}{2}} \quad . \qquad (5\text{-}51b)$$

By eliminating the first space derivative between Equations (5-51a) and (5-51b), we obtain

$$\frac{0.4343\ C_c^{\ell-1}A}{(1+e_{\ell-1})}\ \frac{\partial h}{\partial T} - R_{\ell-1}^2(\overline{p}_o + p - h^*)\ \frac{2k^*_{\ell-1}}{\Delta Z^2}\ (h_{i-1,j+\frac{1}{2}} - h_{i,j+\frac{1}{2}}) =$$

$$\frac{R_{\ell-1}k^*_{\ell}}{R_{\ell}k^*_{\ell-1}}\ \frac{\left[\ \frac{2k^*_{\ell-1}}{\Delta Z} + \frac{\partial k^*_{\ell-1}}{\partial Z}\ \right]}{\left[\ -\frac{2k^*_{\ell}}{\Delta Z} + \frac{\partial k^*_{\ell}}{\partial Z}\ \right]}\left\{\ \frac{0.4343\ C_c^{\ell}A}{(1+e_{\ell})}\ \frac{\partial h}{\partial T} - R_{\ell}^2(\overline{p}_o + p - h^*)\cdot\right.$$

$$\left.\frac{2k^*_{\ell}}{\Delta Z^2}\ (h_{i+1,j+\frac{1}{2}} - h_{i,j+\frac{1}{2}})\ \right\} \quad , \qquad (5\text{-}52)$$

which can be reduced to

$$0.4343\ A\left[\frac{C_c^{\ell-1}}{1+e_{\ell-1}} - \rho\frac{C_c^{\ell}}{1+e_{\ell}}\right]\frac{\partial h}{\partial T} + \frac{2\ (\bar{p}_o + p - h^*)}{\Delta Z^2}\left[R^2_{\ell-1}k^*_{\ell-1} - \rho\, R^2_{\ell}k^*_{\ell}\right].$$

$$h_{i,j+\frac{1}{2}} - \frac{2\ R_{\ell-1}k^*_{\ell-1}}{\Delta Z^2}\ (\bar{p}_o + p - h^*)\ h_{i-1,j+\frac{1}{2}} + \rho\frac{2\ R_{\ell}k^*_{\ell}}{\Delta Z^2}\ (\bar{p}_o + p - h^*)\ .$$

$$h_{i+1,j+\frac{1}{2}} = 0 \quad , \tag{5-53}$$

where the interface factor, ρ, is defined as

$$\rho = \frac{R_{\ell-1}k^*_{\ell}}{R_{\ell}k^*_{\ell-1}}\frac{\left[\frac{2\ k^*_{\ell-1}}{\Delta Z} + \frac{\partial k^*_{\ell-1}}{\partial Z}\right]}{\left[-\frac{2\ k^*_{\ell}}{\Delta Z} + \frac{\partial k^*_{\ell}}{\partial Z}\right]} \quad . \tag{5-54}$$

Finally, use of the Crank-Nicolson scheme on Equation (5-53) yields

$$0.4343\ A\left[\frac{C_c^{\ell-1}}{1+e_{\ell-1}} - \rho\frac{C_c^{\ell}}{1+e_{\ell}}\right]\left[\frac{h_{i,j+1} - h_{i,j}}{\Delta T}\right] + \frac{\bar{p}_o + p - h^*}{\Delta Z^2}\left[R^2_{\ell-1}k^*_{\ell-1} -\right.$$

$$\left.\rho\, R^2_{\ell}k^*_{\ell}\right](h_{i,j+1} + h_{i,j}) - \frac{\bar{p}_o + p - h^*}{\Delta Z^2}\ R_{\ell-1}k^*_{\ell-1}\ (h_{i-1,j+1} + h_{i-1,j}) +$$

$$\rho\frac{\bar{p}_o + p - h^*}{\Delta Z^2}\ R_{\ell}k^*_{\ell}\ (h_{i+1,j+1} + h_{i+1,j}) = 0 \quad . \tag{5-55}$$

The same procedure is followed if the material at the boundary is partially saturated, but the Taylor expansions are given by Equations (5-47a) and (5-47b), the boundary condition by Equation (5-22), and the governing field equation by Equation (5-13); the resulting difference equation is

$$A\,(C^*_{\ell-1} - \rho\, C^*_{\ell}) \left[\frac{h_{i,j+1} - h_{i,j}}{\Delta T} \right] + \frac{1}{\Delta Z^2} \left[R^2_{\ell-1} k^*_{\ell-1} - \rho\, R^2_{\ell} k^*_{\ell} \right] (h_{i,j+1} + h_{i,j}) +$$

$$\frac{\rho}{\Delta Z^2} R^2_{\ell} k^*_{\ell} (h_{i+1,j+1} + h_{i+1,j}) - \frac{1}{\Delta Z^2} R^2_{\ell-1} k^*_{\ell-1} (h_{i-1,j+1} + h_{i-1,j}) -$$

$$\left[R_{\ell-1} \frac{\partial k^*_{\ell-1}}{\partial Z} - \rho \frac{\partial k^*_{\ell}}{\partial Z} R_{\ell} \right] - R_{\ell-1} \left[\frac{2\, k^*_{\ell-1}}{\Delta Z} + \frac{\partial k^*_{\ell-1}}{\partial Z} \right] \left[\frac{k^*_{\ell}}{k^*_{\ell-1}} - 1 \right] = 0 \ , \quad (5\text{-}56)$$

where the interface factor, ρ, is defined by Equation (5-54). Note that the matrix row formed by the coefficients of Equation (5-55) or (5-56) complies with the banded characteristic of the general matrix. By using N-1 equations of this form (one for each interface point), we obtain a system of Nn+1 algebraic equations that will be solved by Gaussian elimination, and the solution of this system of equations will constitute an approximation to the solution of the continuous boundary value problem.

STABILITY AND CONVERGENCE

In this section a bound on the maximum absolute value of solutions to the difference equations approximating Equations (5-31) and (5-32) will be established, the conditions necessary for this bound to exist will be discussed, and it will be proven that the existence of this bound implies stability and convergence. Consider the parabolic differential equation

$$\frac{\partial \phi}{\partial t} = a(z,t) \frac{\partial^2 \phi}{\partial z^2} + 2\, b(z,t) \frac{\partial \phi}{\partial z} + c(z,t) \quad (5\text{-}57)$$

where $a(z,t) > 0$ for all values of z and t. As a consequence of the linearization technique that has been used, Equations (5-31) and (5-32) both have the same general form as Equation (5-57). The application of a numerical method to Equation (5-57) means that, at a certain number of

points in the integration domain (grid points), the value of the function $\phi(z,t)$ will be approximated by a quantity $\varphi(z,t)$ which is the solution of a system of algebraic equations called difference equations. In particular, if the Crank-Nicolson scheme is used, the system of difference equations approximating Equation (5-57) is given by

$$\frac{\varphi_{i,j+1} - \varphi_{i,j}}{\Delta t} = a\left[\frac{\varphi_{i+1,j+\frac{1}{2}} - 2\varphi_{i,j+\frac{1}{2}} + \varphi_{i-1,j+\frac{1}{2}}}{\Delta z^2}\right] + 2b\left[\frac{\varphi_{i+1,j+\frac{1}{2}} - \varphi_{i-1,j+\frac{1}{2}}}{2\Delta z}\right] + c \quad , \tag{5-58}$$

which, by use of $\lambda = \Delta t / \Delta z^2$, may be written

$$\varphi_{i,j+1} - \varphi_{i,j} = a\lambda(\varphi_{i+1,j+\frac{1}{2}} - 2\varphi_{i,j+\frac{1}{2}} + \varphi_{i-1,j+\frac{1}{2}}) + b\lambda\Delta z(\varphi_{i+1,j+\frac{1}{2}} - \varphi_{i-1,j+\frac{1}{2}}) + c\Delta t \quad . \tag{5-59}$$

If $\varphi_{i,j+\frac{1}{2}}$ is replaced by $\frac{1}{2}(\varphi_{i,j+1} + \varphi_{i,j})$, Equation (5-59) becomes

$$(1 + a\lambda)\varphi_{i,j+1} = \lambda(a + b\Delta z)\varphi_{i+1,j+\frac{1}{2}} + (1 - a\lambda)\varphi_{i,j} + \lambda(a - b\Delta z)\varphi_{i-1,j+\frac{1}{2}} + c\Delta t \quad . \tag{5-60}$$

Subject to the conditions given by

$$(1 - a\lambda) \geq 0 \; ; \quad z \in D, \quad t \geq 0 \quad , \tag{5-61a}$$

$$a - \Delta z\,|b| \geq 0 \; ; \quad z \in D, \quad t \geq 0 \quad , \tag{5-61b}$$

where D represents the integration domain, all coefficients of φ in Equation (5-60) are positive. Then, taking absolute values in Equation (5-60) and defining

$$A_j = \max_i |\omega_{i,j}| \tag{5-62a}$$

$$C = \max_i |c| \quad ; \quad t \leq j\Delta t \quad , \tag{5-62b}$$

allows us to write

$$(1 + a\lambda)|\omega_{i,j+1}| \leq \lambda(a + b\Delta z)A_{j+\frac{1}{2}} + (1 - a\lambda)A_j + \lambda(a - b\Delta z)A_{j+\frac{1}{2}} + C\Delta t \quad , \tag{5-63}$$

which can be reduced to

$$(1 + a\lambda)|\omega_{i,j+1}| \leq A_j + a\lambda A_{j+1} + C\Delta t \quad . \tag{5-64}$$

If the maximum occurs at the interior point m, we have

$$A_{j+1} = |\omega_{m,j+1}| \quad , \tag{5-65}$$

which may be introduced into Equation (5-64) to obtain

$$A_{j+1} \leq A_j + C\Delta t \quad , \tag{5-66}$$

and by recurrence we may write

$$A_{j+1} \leq A_{j-1} + 2C\Delta t \leq A_{j-2} + 3C\Delta t \leq \ldots \leq \max|F(z)| + Ct \quad , \tag{5-67}$$

where F(z) is the initial condition. Therefore, it has been proved that, if Equations (5-61) apply, the approximate solution is bounded at any interior point. If the boundary conditions are of the Dirichlet type, the solution is bounded at the boundary and the bound is the boundary value at each time. If the boundary condition is of the mixed type, namely

$$\frac{\partial \phi}{\partial z} + p\,\phi = q \tag{5-68}$$

and if the Taylor expansion technique is used to approximate the solution at the boundary, the resulting difference equation for the lower boundary ($z = 0$) is

$$\varphi_{1,j+1} = \varphi_{1,j} + a\lambda(\varphi_{2,j} + \varphi_{2,j+1} - \varphi_{1,j} - \varphi_{1,j+1} + 2q\Delta z - p\Delta z\varphi_{1,j} - p\Delta z\varphi_{1,j+1}) + 2b\Delta t\{q - \frac{p}{2}(\varphi_{1,j} + \varphi_{1,j+1})\} + c\Delta t \quad , \tag{5-69}$$

which may be rewritten as

$$(1 + a\lambda + a\lambda p\Delta z + bp\Delta t)\varphi_{1,j+1} = (1 - a\lambda - a\lambda p\Delta z - bp\Delta t)\varphi_{1,j} + a\lambda\varphi_{2,j} + a\lambda\varphi_{2,j+1} + 2a\lambda q\Delta z + 2bq\Delta t + c\Delta t \quad . \tag{5-70}$$

Furthermore, if

$$(1 + a\lambda + a\lambda p\Delta z + bp\Delta t) \geq 0 \; ; \quad z = 0 \tag{5-71a}$$

and

$$(1 - a\lambda - a\lambda p\Delta z - bp\Delta t) \geq 0 \; ; \quad z = 0, \tag{5-71b}$$

absolute values may be taken in Equation (5-70) to obtain

$$(1 + a\lambda + a\lambda p\Delta z + bp\Delta t)|\varphi_{1,j+1}| \leq (1 - a\lambda - a\lambda p\Delta z - bp\Delta t)|\varphi_{1,j}| + a\lambda|\varphi_{2,j}| + a\lambda|\varphi_{2,j+1}| + D \quad , \tag{5-72}$$

where $D = |2a\lambda q\Delta z + 2bq\Delta t + c\Delta t|$ is a finite constant. Since the maximum is assumed to lie on the boundary, that is, $|\varphi_{2,j+1}| \leq |\varphi_{1,j+1}|$, the definition

$$A_j = \max\{|\varphi_{1,j}|, \; |\varphi_{2,j}|\} \tag{5-73}$$

yields

$$(1 + a\lambda p\Delta z + bp\Delta t)|\varphi_{1,j+1}| < (1 - a\lambda p\Delta z - bp\Delta t)A_j + D \quad , \tag{5-74}$$

which means that $|\varphi_{1,j+1}|$ is bounded, because if $A_j = |\varphi_{2,j}|$, A_j is bounded by Equation (5-67), and if $A_j = |\varphi_{1,j}|$, A_j is bounded by recurrence. Use of the same procedure allows it to be proven that, if

$$(1 + a\lambda - a\lambda p\Delta z + bp\Delta t) \geq 0 \; ; \quad z = n \tag{5-75a}$$

and

$$(1 - a\lambda + a\lambda p\Delta z - bp\Delta t) \geq 0 \; ; \quad z = n \quad , \tag{5-75b}$$

the boundary value at the upper surface is bounded by

$$(1 - a\lambda p\Delta z + bp\Delta t)|\varphi_{n,j+1}| < (1 + a\lambda p\Delta z - bp\Delta t)A_j + D \quad , \tag{5-76}$$

where in this case $A_j = \max\{|\varphi_{n,j}|, |\varphi_{n-1,j}|\}$ and $D = |-2a\lambda q\Delta z + 2bq\Delta t + c\Delta t|$.

It will now be proven that the existence of this maximum principle implies stability and convergence, if and only if, the difference approximation is consistent with the differential equation. A difference approximation , $L_{i,j}(\phi) = c$, is said to be consistent with a differential equation, $L(\phi) = c$, if

$$\lim_{\Delta z, \Delta t \to 0} |L(\phi) - L_{i,j}(\phi)| = 0 \quad . \tag{5-77}$$

Expansion of $\phi_{i,j}$ and $\phi_{i,j+1}$ in a Taylor series gives

$$\phi_{i,j} = \phi_{i,j+\frac{1}{2}} - \frac{\Delta t}{2}\frac{\partial \phi}{\partial t}\bigg|_{i,j+\frac{1}{2}} + \frac{(\Delta t)^2}{4}\frac{\partial^2 \phi}{\partial t^2}\bigg|_{i,j+\frac{1}{2}} - \frac{(\Delta t)^3}{24}\frac{\partial^3 \phi}{\partial t^3}\bigg|_{i,\xi} \tag{5-78}$$

and

$$\phi_{i,j+1} = \phi_{i,j+\frac{1}{2}} + \frac{\Delta t}{2}\left.\frac{\partial \phi}{\partial t}\right|_{i,j+\frac{1}{2}} + \frac{(\Delta t)^2}{4}\left.\frac{\partial^2 \phi}{\partial t^2}\right|_{i,j+\frac{1}{2}} + \frac{(\Delta t)^3}{24}\left.\frac{\partial^3 \phi}{\partial t^3}\right|_{i,\tau} , \quad (5\text{-}79)$$

where ξ and τ represent intermediate times. Subtracting Equation (5-79) from Equation (5-78), we get

$$\left.\frac{\partial \phi}{\partial t}\right|_{i,j+\frac{1}{2}} - \frac{1}{\Delta t}(\phi_{i,j+1} - \phi_{i,j}) = \frac{(\Delta t)^2}{24}\left[\left.\frac{\partial^3 \phi}{\partial t^3}\right|_{i,\xi} + \left.\frac{\partial^3 \phi}{\partial t^3}\right|_{i,\tau}\right] = O\{(\Delta t)^2\}. \quad (5\text{-}80)$$

By use of the same procedure, it can be easily shown that

$$\left.\frac{\partial \phi}{\partial z}\right|_{i,j+\frac{1}{2}} - \frac{1}{2\Delta z}(\phi_{i+1,j+\frac{1}{2}} - \phi_{i-1,j+\frac{1}{2}}) = O\{(\Delta z)^2\} \quad (5\text{-}81)$$

and

$$\left.\frac{\partial^2 \phi}{\partial z^2}\right|_{i,j+\frac{1}{2}} - \frac{1}{(\Delta z)^2}(\phi_{i+1,j+\frac{1}{2}} - 2\phi_{i,j+\frac{1}{2}} + \phi_{i-1,j+\frac{1}{2}}) = O\{(\Delta z)^2\} . \quad (5\text{-}82)$$

Therefore, subtracting the Crank-Nicolson approximation given by Equation (5-58) from Equation (5-57) yields

$$L(\phi) - L_{i,j}(\phi) = O\{(\Delta t)^2\} - aO\{(\Delta z)^2\} - 2bO\{(\Delta z)^2\} =$$

$$O\{(\Delta t)^2\} + O\{(\Delta z)^2\} = E_{i,j} , \quad (5\text{-}83)$$

which shows that the approximation is indeed consistent with the differential equation. The error $E_{i,j} = O\{(\Delta t)^2\} + O\{(\Delta z)^2\}$ is called truncation error, and it is inherent to the discretization technique.

In addition to consistency, a difference equation is said to be convergent if the approximate solution, $\varphi_{i,j}$, satisfies

$$\lim_{\Delta z, \Delta t \to 0} |\phi_{i,j} - \varphi_{i,j}| = 0 . \quad (5\text{-}84)$$

Let $\epsilon_{i,j}$ be the error defined by

$$\epsilon_{i,j} = \varpi_{i,j} - \phi_{i,j} \quad . \tag{5-85}$$

Subtracting Equation (5-57) from Equation (5-58) and changing the notation to differential and difference operators yields

$$L_{i,j}(\varpi) - L(\phi) = 0 \quad , \tag{5-86}$$

which, with appropiate mathematical manipulation and consideration for the linearity of the operators, can be written as

$$0 = L_{i,j}(\varpi) - L(\phi) + L_{i,j}(\phi) - L_{i,j}(\phi) =$$

$$L_{i,j}(\varpi-\phi) - \{L(\phi) - L_{i,j}(\phi)\} \quad . \tag{5-87}$$

The definition of the truncation error given by Equation (5-83) allows Equation (5-87) to be expressed as

$$L_{i,j}(\epsilon_{i,j}) = E_{i,j} \quad . \tag{5-88}$$

Therefore, the error, $\epsilon_{i,j}$, satisfies a system of difference equations identical to the one represented by Equation (5-58), where c is replaced by E; this means that the error is bounded. Then, if the grid spacing and the characteristics of the differential equation satisfy Equations (5-61), Equation (5-67) gives

$$|\epsilon_{i,j}| \leq \max|F(\epsilon)| + t \max|E_{i,j}| \quad . \tag{5-89}$$

Since the initial error, $F(\epsilon)$, is zero, we have

$$|\epsilon_{i,j}| \leq t \max|E_{i,j}| \quad , \tag{5-90}$$

and therefore ϵ can be made as small as necessary by decreasing the truncation error (that is, reducing the grid spacing). Thus, the difference scheme does converge, and this convergence implies stability (that is, the small error introduced in the nonhomogeneous term remains bounded as time increases) if, and only if, the difference equations are consistent (Douglas, 1956; Keller, 1967). In summary, the consistency, convergence, and stability conditions are represented by Equations (5-61), (5-71), and (5-75), respectively. In the case under consideration throughout this study, application of these conditions results in the following relationships.

Saturated Flow

Interior points:

$$1 - \frac{\Delta T}{(\Delta Z)^2} R_\ell^2 (\overline{P}_o + p - h^*) \frac{k^*(1+e)}{0.4343 C_c A} \geq 0 \quad , \tag{5-91a}$$

$$k^* - \Delta Z \left| \frac{\partial k^*}{\partial Z} \right| \geq 0 \quad . \tag{5-91b}$$

Lower boundary, impervious surface:

$$1 - \frac{\Delta T}{(\Delta Z)^2} (\overline{P}_o + p - h^*) \frac{k^*(1+e)}{0.4343 C_c A} \geq 0 \quad . \tag{5-92}$$

Lower boundary, impeded drainage (impedance factor = λ):

$$1 + \frac{\Delta T}{(\Delta Z)^2} (\overline{P}_o + p - h^*) \frac{k^*(1+e)}{0.4343 C_c A} (1 - \Delta Z \lambda H_1) -$$

$$\Delta T (\overline{P}_o + p - h^*) \frac{(1+e)}{0.4343 C_c A} \frac{\partial k^*}{\partial Z} \lambda H_1 \geq 0 \quad , \tag{5-93a}$$

$$1 - \frac{\Delta T}{(\Delta Z)^2} (\bar{p}_o + p - h^*) \frac{k^*(1+e)}{0.4343 C_c A} (1 - \Delta Z \lambda H_1) +$$

$$\Delta T (\bar{p}_o + p - h^*) \frac{(1+e)}{0.4343 C_c A} \frac{\partial k^*}{\partial Z} \lambda H_1 \geq 0 \quad . \tag{5-93b}$$

Partially Saturated Flow

Interior points:

$$1 - \frac{\Delta T}{(\Delta Z)^2} \frac{R_\ell^2 k^*}{C^* A} \geq 0 \quad , \tag{5-94a}$$

$$k^* - \Delta Z \left| \frac{\partial k^*}{\partial Z} \right| \geq 0 \quad . \tag{5-94b}$$

Upper boundary, evapotranspiration condition:

$$1 - \frac{\Delta T}{(\Delta Z)^2} \frac{R_\ell^2 k^*}{C^* A} \geq 0 \quad . \tag{5-95}$$

All other boundary conditions are consistent and bounded by definition.

In the convergence conditions given by Equations (5-91a), (5-92), (5-93a), (5-93b), (5-94a), and (5-95), the reduced time factor, A, will affect the numerical result of the inequality. Therefore, if the scheme begins to lose convergence due to the nonfullfilment of one of these conditions, the problem can be automatically corrected by changing the value of the reduced time factor, and thus changing the real time increment, but without modifying the dimensionless time step or any data values.

SUMMARY

A numerical technique has been developed to solve approximately the nonlinear boundary value problem defined in Chapter 3. The governing differential equations and the boundary conditions have been reduced to a dimensionless form, and a parameter, A, termed the reduced time factor, has been introduced in the definition of the dimensionless time variable. This latter factor is useful because (a) it allows the real time step to be modified without changing the dimensionless time increment (thereby causing little effect on the truncation error), and (b) it enables the program to be written so that the reduced time factor will be automatically changed if some convergence problem is identified by the test equations.

The Crank-Nicolson approach to the finite difference technique has been used, and the Neumann and mixed boundary conditions have been analyzed in such a way that both the governing equation and the surface condition are applied at the points on the boundary. The continuity condition at the interface between layers has been studied in a similar form. As a consequence of the technique used to discretize the boundary conditions, the matrix of coefficients remains banded, and a non-iterative Gaussian elimination method can be used to solve the resulting system of linear algebraic equations. Finally, the consistency, convergence, and stability conditions have been analyzed and explicitely obtained, thereby providing a testing and automodifying mechanism to check for stability problems.

CHAPTER 6

SYNTHESIS OF RESULTS

Presented in this chapter are some results determined by use of (a) the numerical solution outlined in Chapter 5 for different boundary conditions and (b) the empirical relationships given in Chapter 4 for the engineering properties of typical dredging wastes. The calculated settlement versus time curves are compared with actual field measurements of settlement at various locations in the Penn 7 containment area in Toledo, Ohio. A study is undertaken to assess the effect of a change in boundary conditions on the settlement rate of the deposit and the circumstances under which artificially improved boundary conditions might be justified.

DESCRIPTION OF FIELD PROGRAM

The accuracy of the settlements calculated by the mathematical model was evaluated by direct comparison with corresponding settlements measured at various points in the Penn 7 containment area in Toledo, Ohio. This site, the plan view of which is shown in Figure 6-1, is located on the northern bank of the Maumee River and has an area of about 1.3 million square feet (0.12 million square meters). The dredging slurry was pumped into the site through an inflow pipe, represented in Figure 6-1 by point A, and the excess water was returned to the river through an overflow weir situated at point B. The site was essentially filled during two dredging seasons; the first season extended from August 28, 1972 to December 19, 1972, and a total bin-measure volume of 0.57 million cubic yards (0.44 million cubic meters) was deposited; the second season began

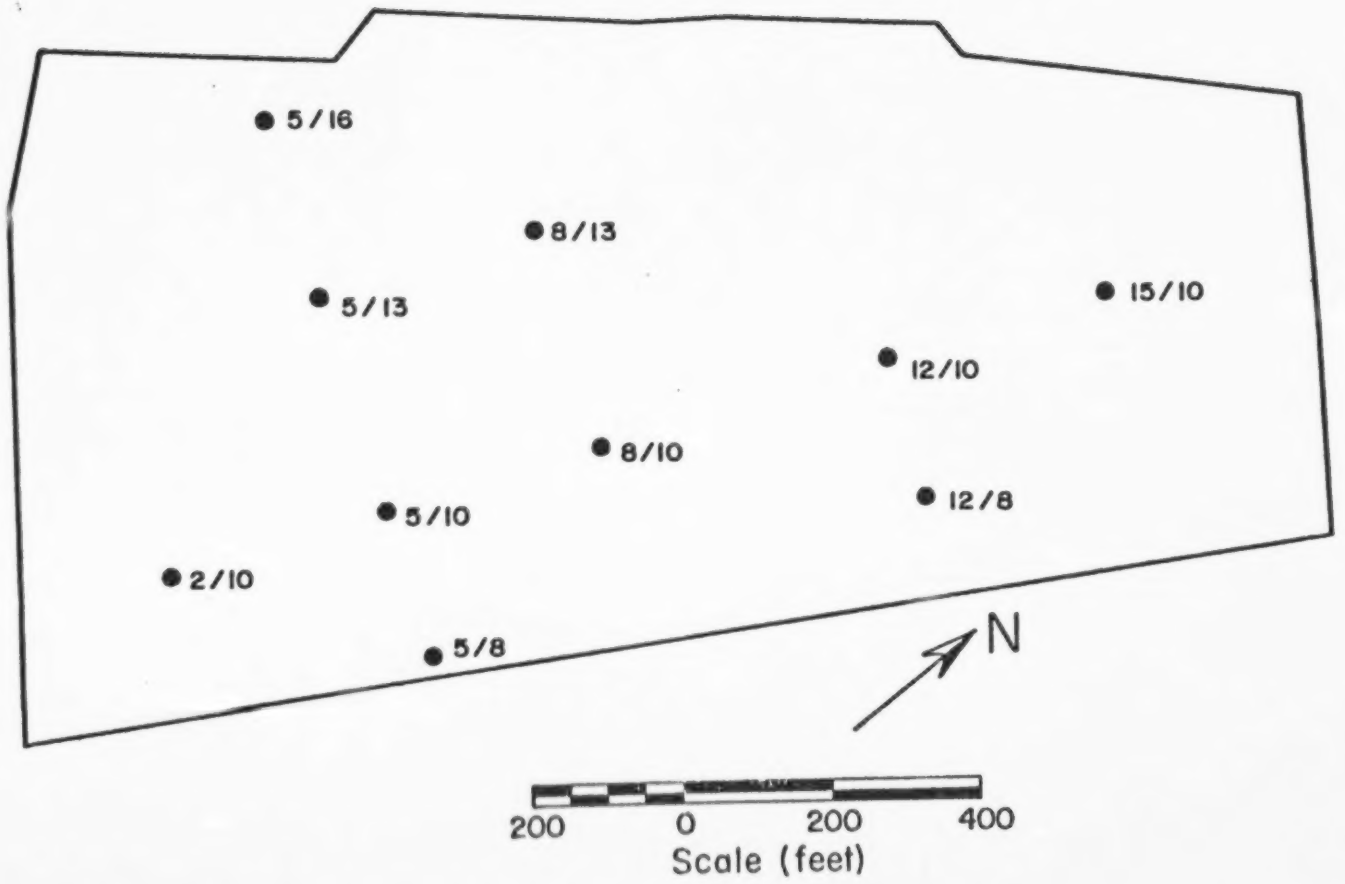

Figure 6-1. General Plan of Penn 7 and Location of Settlement Plates

on June 11, 1973 and ended on August 20, 1973, and a total bin-measure volume of 0.35 million cubic yards (0.27 million cubic meters) was pumped into the area.

If a coefficient of 0.65 is used to relate the disposal-site volume to the bin-measure volume (Krizek and Giger, 1974), the average thickness of the dredged material will be 2.35 meters for the first cycle and 1.44 meters for the second cycle. Although the thickness of each layer is not constant throughout the site (the granular material was deposited in thicker layers near the inflow pipe, whereas the finer grained materials are found in thinner layers near the overflow weir), the assumption of a one-dimensional process at any given point provides a reasonable first approximation to the actual conditions. It was further assumed that 5 centimeters of water overlaid the surface of the sediments at the end of each cycle, and this water had to evaporate before any desiccation could take place; this value of 5 centimeters represents the approximate average depth of water that was observed in this area. The thickness of this assumed depth of water will obviously affect the time at which the dredging layer will begin to desiccate.

Settlements in each layer were obtained by means of settlement plates at the bottom and top of the first layer and top of the second layer. Although settlement plates were installed at a number of locations, shown in Figure 6-1, many of these plates were unfortunately destroyed, either by adverse weather conditions or vandalism (primarily the latter). As a result, complete records are available only at four locations (5/8, 5/10, 5/13, and 15/10), and the accuracy of the data for location 15/10 is suspect.

SETTLEMENT-TIME RESPONSE

Once the settlement plates were installed, periodic surveys were conducted to monitor the settlement-time response of the sediments. Figure 6-2 gives the results of these surveys for locations 5/8, 5/10, and 5/13. The time origin in these figures is June 13, 1973, the day on which the settlement plates were installed on the surface of the first layer. The observed layer thickness differ from the average used in the numerical solution by values up to 20%. Hence, as illustrated in Figure 6-2, the observed settlements were normalized in a linear manner to account for these differences. While this method of correction may involve some error (settlements are not necessarily proportional to the thickness of the layer), this error should be relatively small.

The soil characteristics used with the mathematical model were described in Chapter 4; therefore, the general applicability of all cases analyzed herein is restricted by the nature of the dredged materials, namely, the materials should be similar to those encountered in the vicinity of Toledo, Ohio. In addition, since no data were available to identify vertical variations in engineering properties at a given location in the Penn 7 area and since the available data from similar nearby sites indicated generally inconclusive trends, the dredged material at a given location was assumed to be homogeneous in the vertical direction. Accordingly, all the scale heterogeneity functions were assumed to be constant and equal to unity (however, other functions can be easily introduced in the corresponding subroutine if and when sufficient information becomes available to define such relationships). Despite this assumption of initial homogeneity, it is clear that the dredgings will be heterogeneous as soon as the consolidation and desiccation processes begin, since variations in void

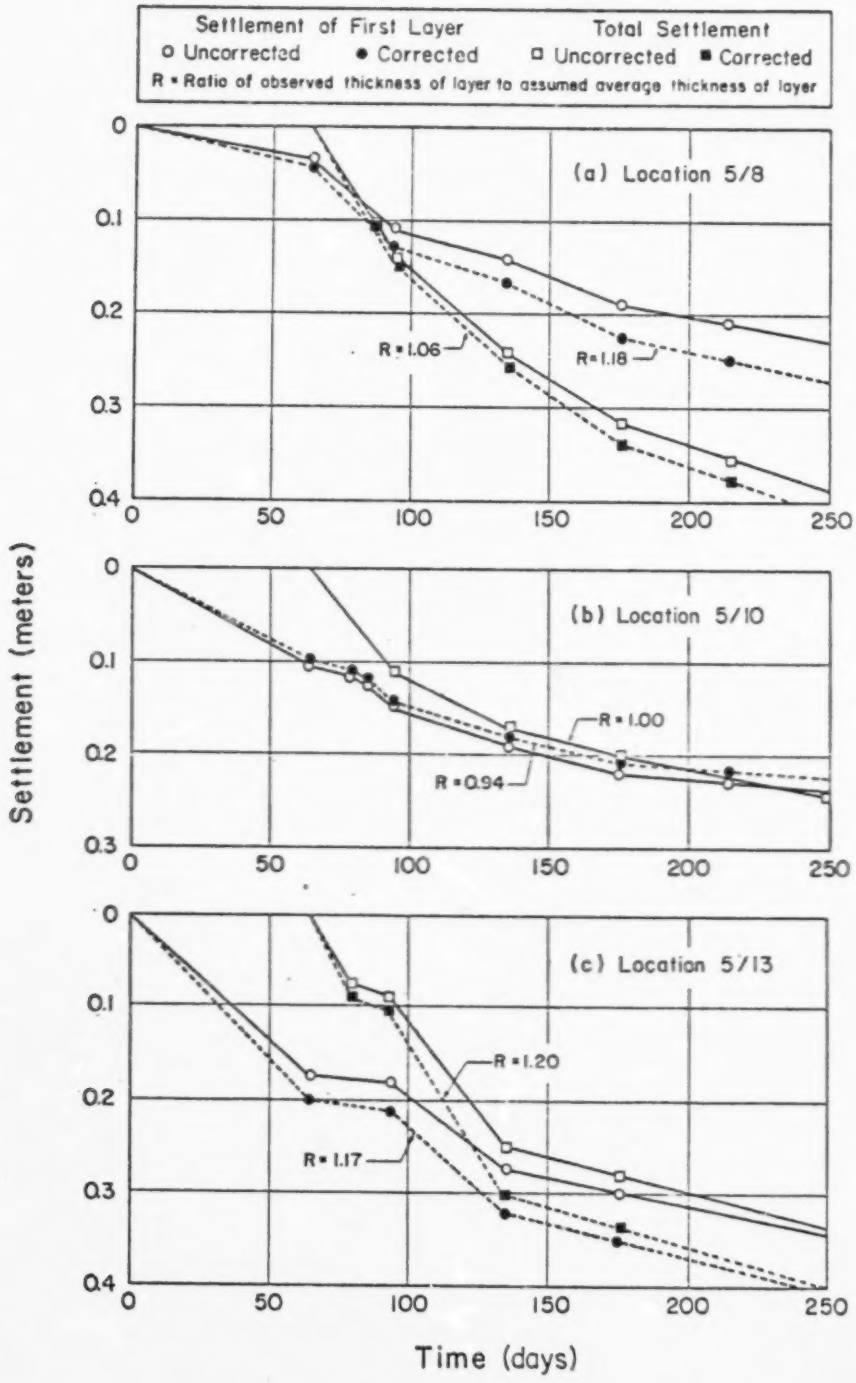

Figure 6-2. Observed Settlements

ratio, water content, and pore water pressure will produce variations in all soil characteristics, such as permeability, compression index, specific water capacity, etc.

EVALUATION OF BOUNDARY CONDITIONS

A number of cases (based on the overall configuration and material characteristics of the Penn 7 site) were studied to evaluate the effect of different boundary conditions on the resulting settlement response, and some of these cases are presented herein. To better appreciate the impact of each boundary condition on the solution, the same statistical year was used in each case to establish the weather conditions. Accordingly, the time necessary for the evaporation of the overlying water will be the same in each case, and the time origin and time scale will be constant.

In each of the settlement versus time curves shown in Figures 6-3 through 6-6, three distinct time zones can be distinguished. The zone extending from the origin to point A represents the desiccation of the first layer; at point A the deposition associated with the second cycle begins, and the zone beyond point B depicts the volume change due to evapotranspiration through the top boundary of the second layer. Throughout this discussion the time origin is February 3, 1973, and points A and B represent June 11, 1973 and September 5, 1973, respectively. From the origin to point B the indicated settlement represents the volume change of the first layer of sediments, whereas beyond point B it represents the volume change of both layers (that is, settlements at the top of the deposit). This means that calculated settlements prior to point B must be compared with the first layer measurements given in Figure 6-2, while comparisons

beyond point B must be made with the total settlement data. Due to the fact that the initial settlement plates at the top of the first layer were placed on June 13, 1973, the comparisons between calculated and observed settlements must be made in terms of relative, rather than absolute, values. Therefore, it is assumed that point A (for the settlement of the first layer) and point B (for the total settlement) represent values that agree with the measured settlements; the plotted points are the relative values with respect to these initial values.

Variations in Drainage Conditions

The actual drainage conditions of the foundation soils at the Penn 7 site were investigated by means of seventeen field permeability tests that were conducted in the area prior to the deposition of dredged materials. Seven of these tests were shallow infiltration tests (depth of 0.6 to 1 meter), six were restoration tests at a depth of 3 meters, and four were 6-meter deep restoration tests. The shallow infiltration tests gave a minimum permeability value of 3×10^{-7} cm/sec and a maximum value of 6.7×10^{-3} cm/sec with a median of 8.7×10^{-5} cm/sec. These values were 6×10^{-7} cm/sec, 4.8×10^{-3} cm/sec, and 2.15×10^{-4} cm/sec, respectively, for the 3-meter restoration tests, and 8.7×10^{-5} cm/sec, 2.3×10^{-2} cm/sec, and 1.12×10^{-3} cm/sec for the 6-meter restoration tests. In the two locations nearest the three previously mentioned settlement plate locations, the range of permeability was from 2.3×10^{-2} cm/sec for the 6-meter well to 4.3×10^{-6} for the shallow well.

Two soil profiles for this site were made available by the Toledo Field Office of the U. S. Army Corps of Engeneers. One of the borings was made at location 5/10 and the other was made about 140 feet (46 meters) south of this location. The profile at location 5/10 shows 11 feet (3.4

meters) of gray sandy clay at the top; since this boring was only 11 feet deep, no information is available on the deeper soils. The other profile is more complicated and shows loose clayey silt for the top 3.4 feet (1.0 meter), soft sandy clay for the next 3.2 feet (1.0 meter), sand for 6.5 feet (2.0 meters), silty sand for 5 feet (1.6 meters), clayey silt for 4.6 feet (1.4 meters), and sand for 21.7 feet (6.6 meters).

The fact that the nature of the foundation soil is quite complex and seems to change very rapidly in a relatively short distance, together with the large range of values obtained for the coefficient of permeability, results in a large variation for possible values of the factor k_f/H_f, where k_f is the coefficient of permeability of the foundation soil and H_f is the thickness of the impeding non-consolidating layer. Therefore, a depth of 15 feet (4.6 meters) was chosen for the underlying non-consolidating layer, and the coefficient of permeability, k_f, was varied to cover the complete range of values of k_f/H_f. Due to the fact that the permeability of the dredged material at the bottom of the deposit changes throughout the consolidation process (decreasing as the void ratio decreases), the impedance factor, λ, given by

$$\lambda = \frac{k_f}{H_f} \frac{1}{k_c} \quad , \tag{6-1}$$

where k_c is the permeability of the dredgings at the bottom surface, is not known. Its initial value ranges from about 60 for $k_f=10^{-4}$ cm/sec to 0.06 for $k_f=10^{-7}$ cm/sec. For values of λ greater than 60 ($k_f>10^{-4}$ cm/sec with $H_f=4.6$ meters), the solution was found to be virtually identical to that obtained by assuming free drainage, while for values of λ less than 0.06 ($k_f<10^{-7}$ cm/sec with $H_f=4.6$ meters) an impervious condition could be assumed.

Figure 6-3 presents some of the results calculated for the various boundary conditions indicated, as well as the measured settlements. Although a comparison of the computed response shown in Figure 6-3 for the different conditions suggests that the nature of the drainage condition at the lower boundary exerts little effect on the settlement versus time response, this is not quite correct, as can be seen in Figure 6-4, which summarizes the four curves shown in Figure 6-3. Curve A in Figure 6-4 corresponds to the assumption of an impervious bottom boundary, curve B to an impeded drainage condition where $k_f=10^{-6}$ cm/sec and $H_f=4.6$ meters, curve C to another impeded drainage condition where $k_f=10^{-5}$ cm/sec and $H_f=4.6$ meters, and curve D represents a free drainage condition at the lower boundary. It can be seen that, although the difference in computed settlements for the different drainage conditions is noticeable during the second stage (namely, when the second layer is being deposited and the situation resembles somewhat a conventional consolidation process), it is much less significant during the first and third stages (namely, during evapotranspiration). This is the case because, during the desiccation process, the pore water pressure gradients near the partially saturated zone are much larger than those near the bottom, and therefore the conditions at the upper boundary exert a significantly greater effect on changes in the pore pressure distribution. Hence, it can be concluded that, while an improvement in the drainage characteristics of the foundation soil will accelerate the settlement rate of the dredgings, this improvement takes place almost exclusively during periods of no evapotranspiration from the soil. Therefore, unless these periods are sufficiently long or frequent, a careful study should be undertaken before any decision is made regarding a costly modification to the drainage conditions at the bottom boundary.

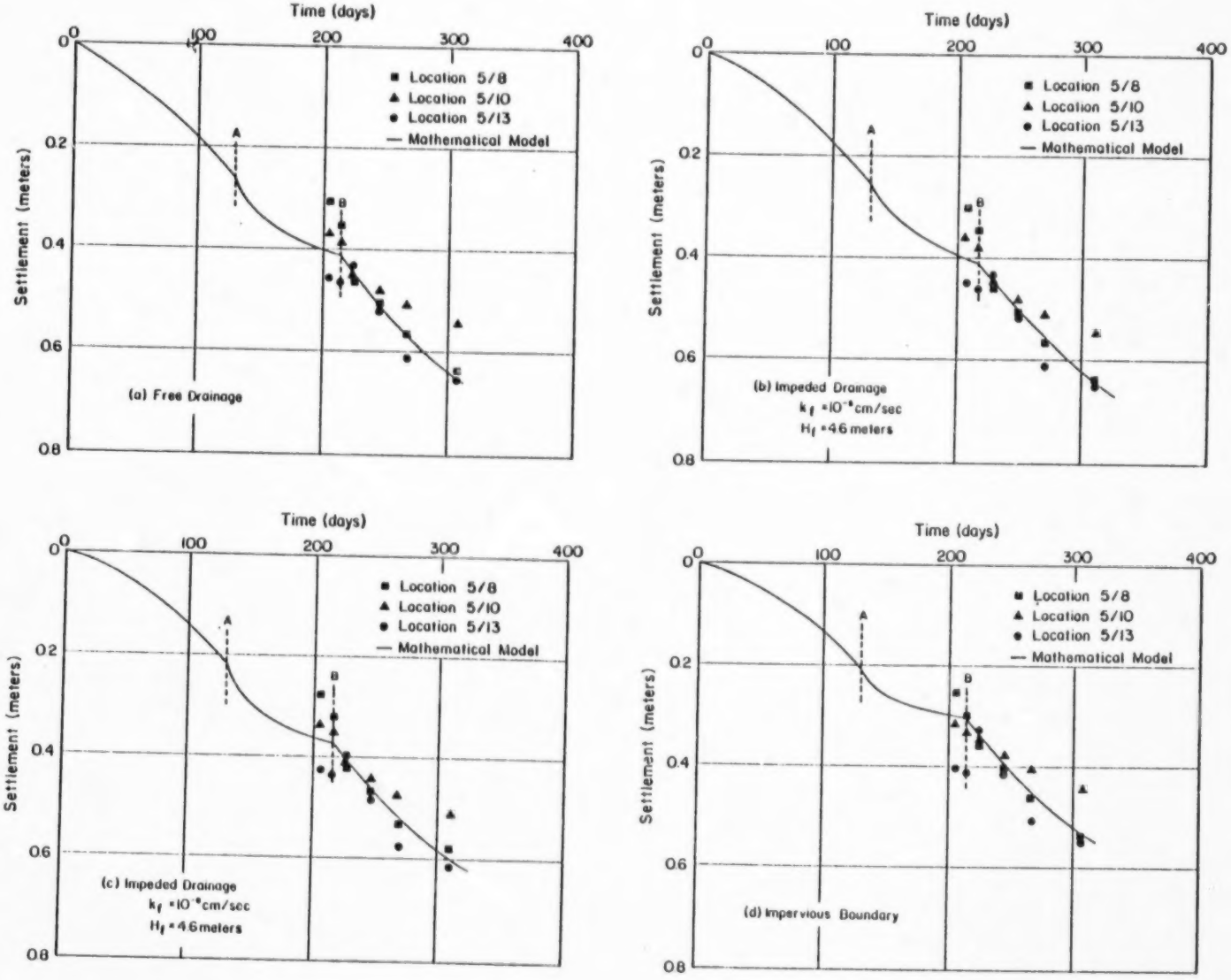

Figure 6-3. Settlement versus Time for No Transpiration and Different Drainage Conditions

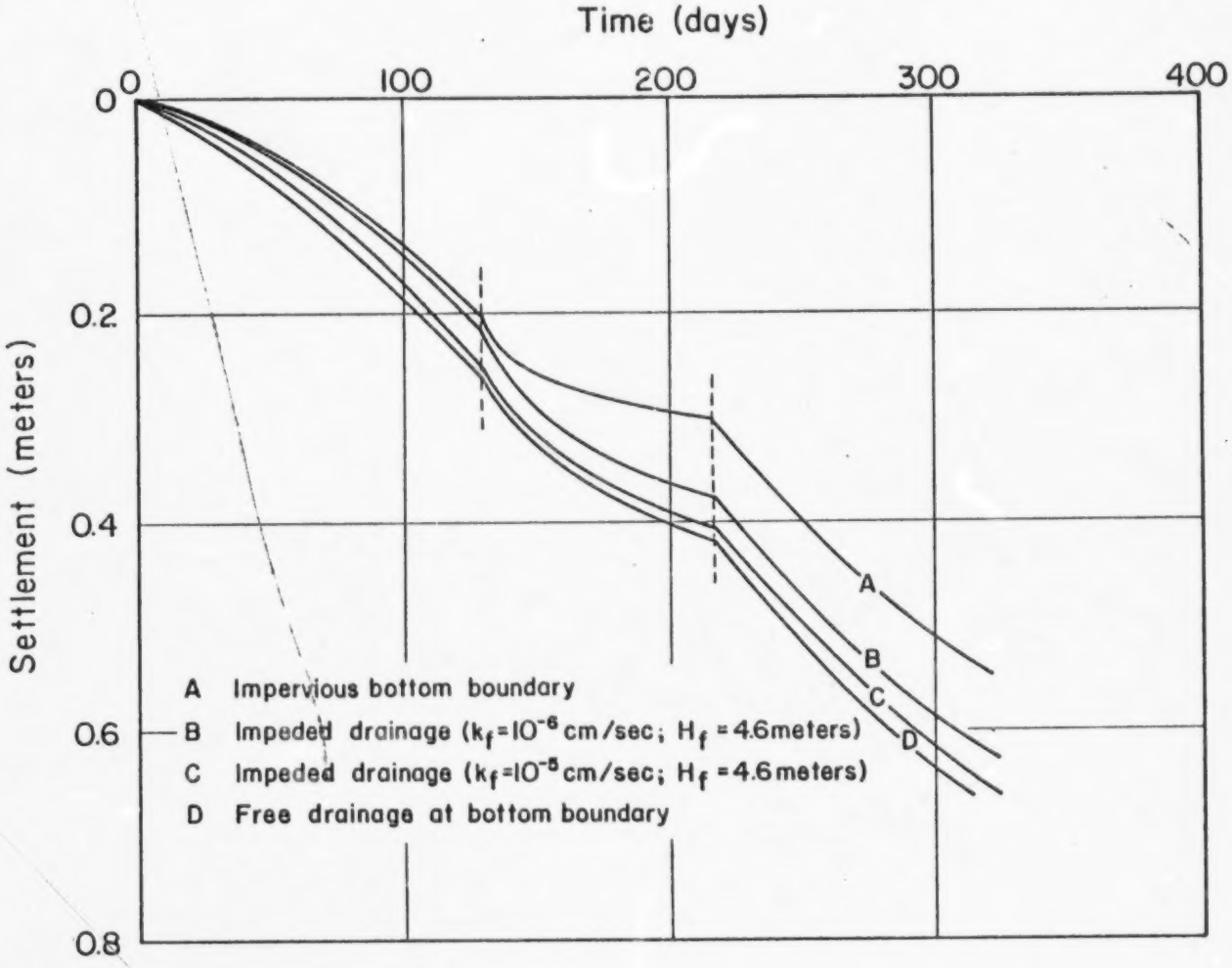

Figure 6-4. Summary of Settlement Response Curves for No Transpiration and Different Drainage Conditions

Variations in Evapotranspiration Conditions

The effect of transpiration on the settlement response of dredged material was investigated by using different coefficients of transpiration, ranging from 0 to 1 (actually increasing the evaporation potential by 100%), in a fixed problem, which was chosen to be the Penn 7 disposal site with an impeded drainage condition ($k_f=10^{-5}$ cm/sec and $H_f=4.6$ meters) at the lower boundary. In the typical results presented in Figure 6-5, it can be observed that the situation is opposite to that found for varying drainage conditions. During periods of no evapotranspiration, the coefficient of transpiration obviously has very little effect on the settlement rate, but it is a very important factor in the first and third stages of the process, as can be observed in Figure 6-6. Although Linsley, Kohler, and Paulhus (1949) have reported that β values greater than 0.75 are not realistic, an upper bound of unity was used in this comparative study to guarantee that the condition of maximum reasonable transpiration has been attained. Since it was found that the settlement response was fairly constant for β values greater than 0.7, the solution obtained for $\beta=1$ can be regarded as an upper limit.

In the application of these coefficients it has been assumed that vegetation starts to grow as soon as the top of the layer becomes slightly unsaturated. Although this may not necessarily be true in all cases, such a condition can be enhanced by seeding the area with properly selected plants (namely, those conducive to rapid growth and high transpiration rates). Results achieved by this method will likely be as good or better than those obtained by improving the drainage conditions at the bottom and the cost will be significantly less. The greater the percentage of last stage desiccation with respect to the total time of deposition, the more important

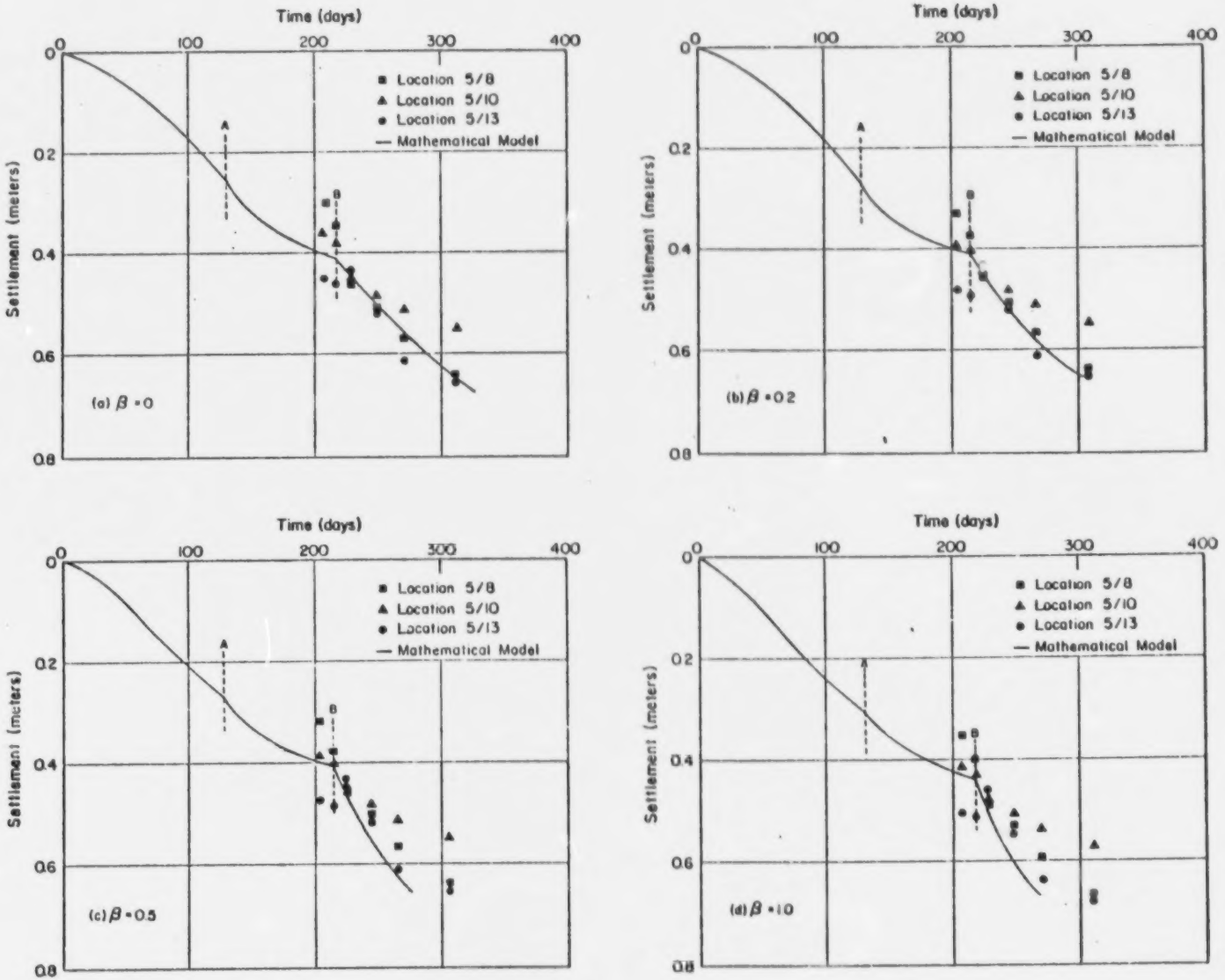

Figure 6-5. Settlement versus Time for Impeded Drainage and Different Coefficients of Transpiration

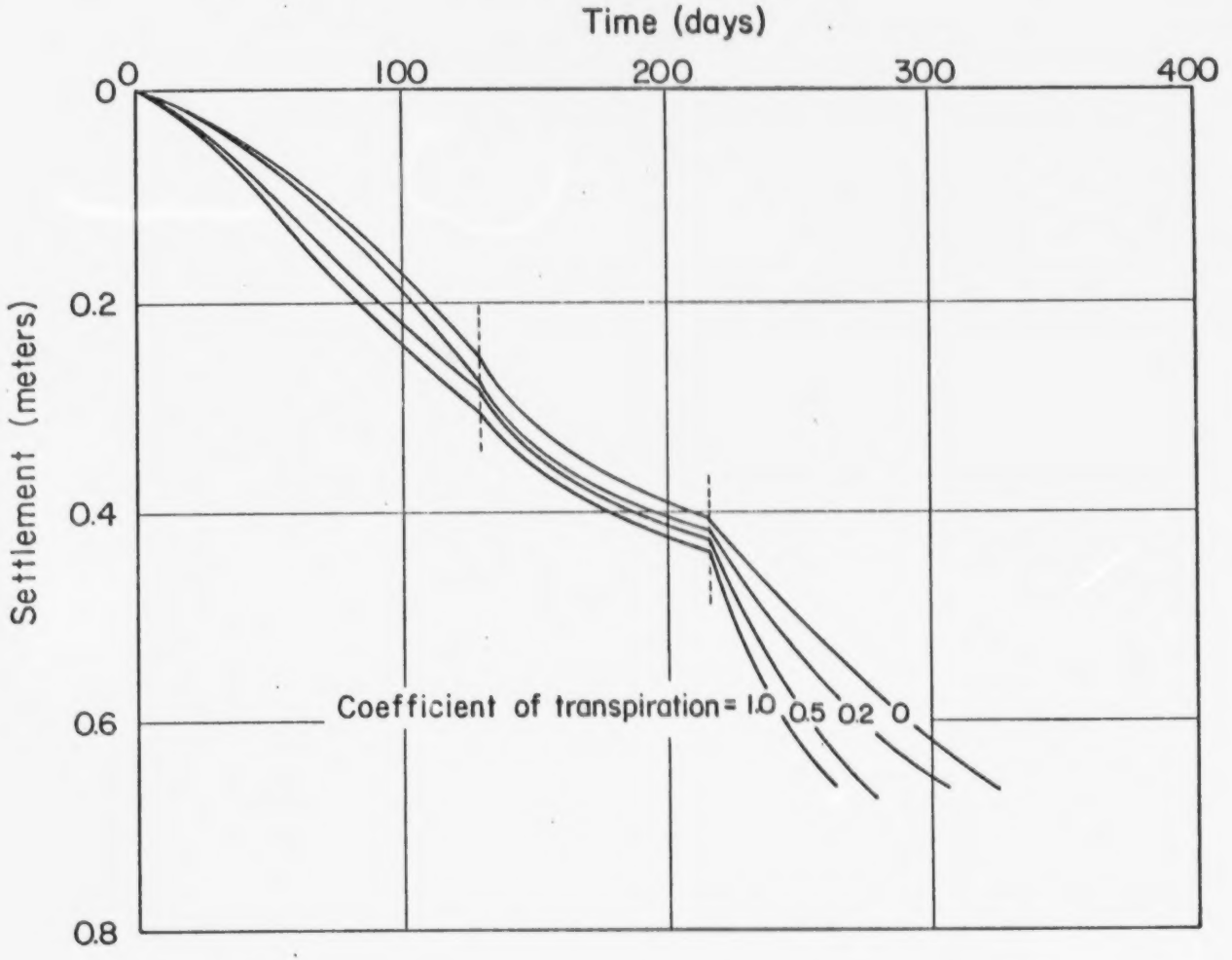

Figure 6-6. Summary of Settlement Response Curves for Impeded Drainage and Different Coefficients of Transpiration

it becomes to artificially increase the coefficient of transpiration. Figure 6-7 shows the actual recorded settlement at Penn 7 and the last stage settlement calculated for various boundary conditions. Curve A represents the case of an impervious bottom surface and no transpiration; curve B, no transpiration and impeded drainage with $k_f=10^{-5}$ cm/sec and $H_f=4.6$ meters; curve C, free bottom drainage and no transpiration; and curves D, E, and F, the same impeded drainage condition as curve B, but with values of 0.2, 0.5 and 1, respectively, for the coefficient of transpiration. The time origin for these curves is September 5, 1973, and the indicated settlements are relative to those developed prior to the beginning of this final stage.

Figure 6-7 illustrates very clearly the effect of transpiration on the settlement response and associated densification of dredged materials in a diked containment area. Suppose, for example, that the impeded drainage represented by curve B governs the drainage at a given point. If the drainage condition at this point were improved to approximate a free drainage boundary, the increase in settlement after 50 days of desiccation would be about 3.5%. However, if a coefficient of transpiration of only 0.2 were achieved, the relative settlement at this time would be increased by about 30%; similarly, improvement would be approximately 50% if the coefficient of transpiration were equal to 0.7.

SUMMARY

The mathematical model described in Chapter 5 and the average soil characteristics outlined in Chapter 4 were applied to study the response of the dredged material placed in the Penn 7 containment area in Toledo, Ohio. A number of cases were investigated to assess the effect of different boundary conditions on the settlement versus time response. A comparison of calculated settlements and measured settlements at three locations in

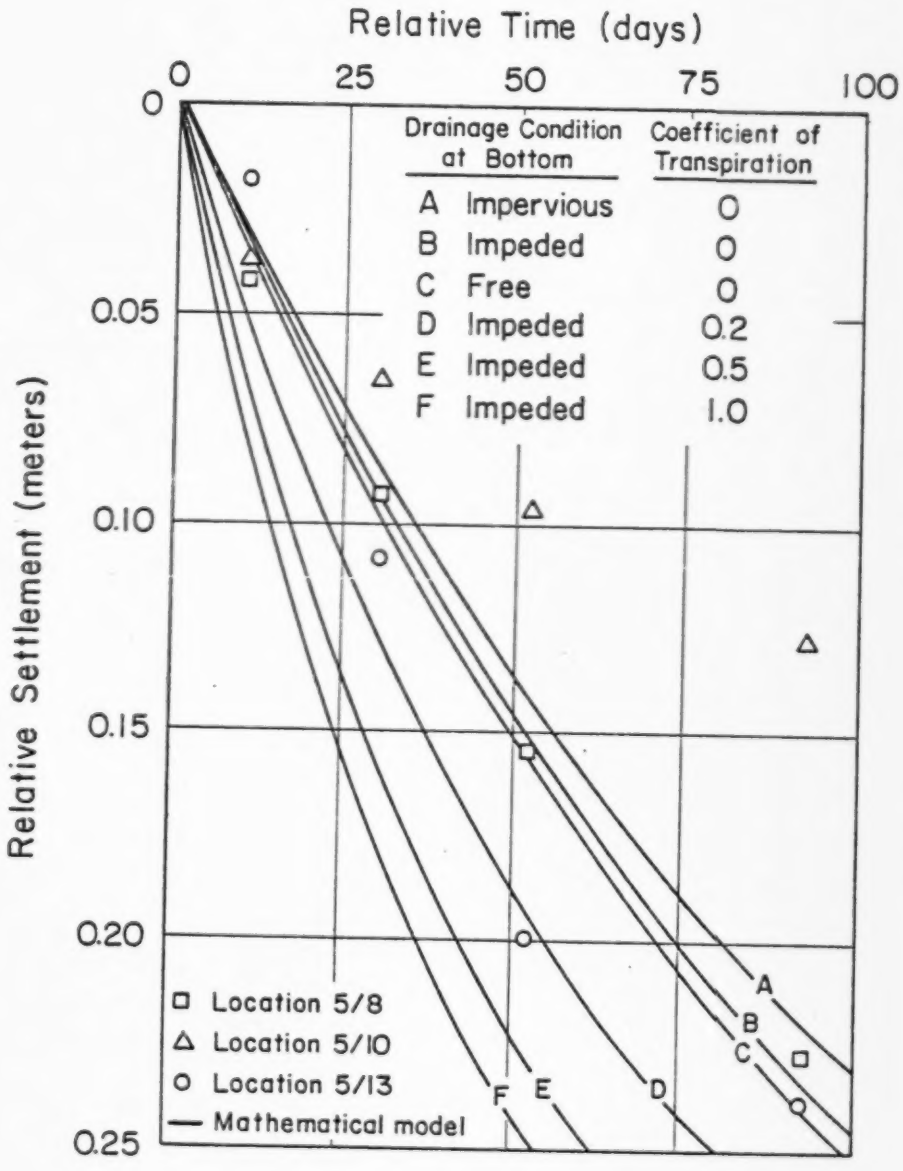

Figure 6-7. Last Stage Settlement Response for Different Boundary Conditions

the area indicates that the calculated values are of the same order of magnitude as the observed values, except at location 5/10 where the calculated values for conditions are no transpiration were significantly larger; although the reasons for this discrepancy have not been clearly identified, it appears that the measured settlement may be in error, since the settlements are nearby locations under similar conditions do agree with calculated values. It was found that drainage conditions at the lower boundary had a greater effect on the amount of settlement during the period prior to desiccation than after desiccation had begun, whereas the opposite is true for the evapotranspiration condition at the upper boundary In view of these results it appears that, unless the dredged materials will be inundated for long periods of time, the cost associated with improving the bottom drainage conditions in order to accelerate the densification of the dredgings should be evaluated very carefully. On the other hand, the use of vegetation with high rates of transpiration seems to offer a feasible method for achieving modest increases in the settlement rate of the fill material.

CHAPTER 7

SUMMARY AND CONCLUSIONS

A mathematical model has been developed to study the desiccation and consolidation of dredged materials that have been deposited in diked containment areas. The differential equation describing saturated or unsaturated flow of water through a consolidating porous medium has been derived in its most general form, and simplifying assumptions have been employed to deduce two nonlinear parabolic differential equations, one for the saturated zone and one for the partially saturated zone. A step-by-step technique has been developed to adjust for the effects of these simplifying assumptions by a multi-stage correction procedure, which includes the differential pressure increase due to the lowering of the water table during desiccation periods or to the deposition of a new layer of sediments during dredgings periods. The governing boundary value problem has been established in terms of pore water pressures and normalized to a dimensionless form.

Based on the results of an extensive experimental program aimed toward defining the engineering properties of typical dredged materials from the vicinity of Toledo, Ohio, empirical relationships have been developed to approximate average values of the permeability, water retention curves, and consolidation characteristics. In cases where experimental data were not available, information reported in the literature and tempered with engineering judgment was employed to make appropriate assumptions.

A finite difference method, in conjunction with a linearization technique, has been used to solve the boundary value problem numerically. The

convergence and stability characteristics of this approximate solution have been studied in considerable detail, and a number of conditions, together with an automatically adjusting coefficient, have been introduced in the computer program to guarantee convergence.

This mathematical model was applied to study the field settlements measured at the Penn 7 disposal site in Toledo, Ohio. The actual consolidation settlements that took place were compared with those calculated by use of the mathematical model, and a parameter study was undertaken to evaluate the effect that changes in the boundary conditions may have on the consolidation and desiccation rates. The lower boundary condition varied through a discrete set of impeded drainage conditions ranging from impervious to free drainage, and the coefficient of transpiration at the upper boundary was changed from 0 (only evaporation) to a maximum of unity (twice the evaporation potential).

Within the context of this study, the following conclusions can be advanced:

1. The rate of consolidation predicted by the mathematical model is approximately the same order of magnitude as most of the actual field observations, and agreement is particularly good during periods of desiccation.

2. Although the drainage condition at the bottom of the sediment layer had a definite effect on the consolidation rate during the period of deposition, its impact during desiccation was almost negligible due to the fact that the dissipation of excess pore pressure is attributed primarily to the evapotranspiration potential.

3. The value of the coefficient of transpiration has an important effect on the consolidation rate during the early stages of desiccation, but this effect tends to disappear as the water table approaches an equilibrium position.

4. Unless the periods of flooding due to deposition of new material are very long or very frequent, it is difficult to justify the cost of improving drainage characteristics at the bottom of the layer.

5. The best way to accelerate the consolidation rate of a landfill composed of dredged spoil, especially after the site has been completely filled, seems to involve the use of vegetation with high transpiration rates and the ability to grow in soils with high water content.

6. The consolidation behavior of dredged materials can apparently not be improved very much with low cost modifications in the boundary conditions.

APPENDIX

GENERAL FEATURES OF COMPUTER PROGRAM

The computer program prepared in this study is completely self-contained and includes a main program and thirteen subroutines, of which seven calculate different soil characteristics, two introduce scale heterogeneity functions, one handles weather data, one generates random numbers, one solves the system of equations, and one performs the end-of-step corrections. Described herein are certain features of the numerical techniques that have been used, the processing time and memory requirements, and a general operational flow chart of the overall mathematical model. The program has been written in FORTRAN IV for the CDC 6400 computer at the Vogelback Computing Center of Northwestern University, and no library subroutines have been used.

NUMERICAL TECHNIQUES

The following numerical techniques have been used throughout the program.

Integration

Except at the boundaries, where the trapezoidal rule was used, Simpson's first and second rule were employed to carry out numerical integration.

Differentiation

Second order two- and three-point difference approximations were used to express derivatives.

Interpolation

In all cases where interpolation was used, a four point Lagrange interpolating polynomial was applied. A large amount of time and effort was de-

voted to testing different interpolation techniques, ranging from Newton's divided-difference interpolating polynomials to high order Chebyshev functions, but it was found that, due to the type of data available in these cases, high order interpolation sometimes resulted in highly irregular distributions with some of the points obviously misplaced. Although the program is prepared to accept numerical data, every effort should be made to introduce soil characteristics as analytical functions or to smooth the data by means of least-square polynomials.

Solution of System of Equations

A Gaussian elimination scheme for a banded matrix of coefficients (band width equal to 3) was employed to solve the resulting system of equations.

Random Number Generation

Random numbers were generated by the multiplicative congruential method, which applies the recursion algorithm

$$x_i = ax_{i-1} \pmod{b}; \quad i = 2,3,\ldots, \qquad \text{(A-1)}$$

to give a random number uniformily distributed between 0 and 1 as

$$y_i = x_i/b ; \quad i = 2,3,\ldots. \qquad \text{(A-2)}$$

In Equation (A-1) a and b are two integer constants such that $b=10^n$ (where n=4 in the case under consideration) and a is close to $2^{n/2}$. If the first value, x_1, is odd and not a multiple of 5, the period before which the numbers begin repeating is b/2.

PROCESSING TIME AND MEMORY REQUIREMENTS

The memory requirements for the maximum program capacity (6 cycles; 40 points per layer) is 74500 (octal) words; this reduces to 53000 (octal) words for 2 cycles and 20 points per layer. To run the program for the problem described in Chater 6 (2 cycles; 20 points per layer; 1250 time steps), about 300 seconds of central processor time with 30 seconds of peripheral processor time were required. The stability conditions described in Chapter 5 will automatically reduce the time step to avoid stability problems; for example, if the ratio k^*/C^* in Equations (5-94a) or (5-95) becomes very large (which may occur for low values of C^*, that is, when the absolute value of the soil-water potential becomes very high), the value of the reduced time factor, A, will increase, thereby decreasing the real time step. Thus, the program could require very long processing times if long periods of desiccation with high values of the evapotranspiration potential are encountered.

As an example, this program was run for the conditions associated with the Riverside disposal area in Toledo, Ohio (this area was filled in five cycles during four different years, but the actual period of deposition for some of the cycles was very short--7 to 10 days), and approximately 2200 seconds of computer time were required to calculate 5529 time steps, corresponding to a real time of 12000 hours and an average real time increment of 2.17 hours (as compared with 6.25 hours for the Penn 7 solution in Chapter 6). If a long and complicated case study is to be conducted, it is very convenient and efficient to stop the program when the position of the water table begins to stabilize and to resume the analysis at a time just prior to the start of the next pumping cycle. Although this will involve some error due to consolidation during the omitted time period, this error

should be small and its value can be reasonably estimated.

OPERATIONAL FLOW CHART

Given in Figure A-1 is a very concise operational flow chart for the overall mathematical model. No attempt is made to describe in detail the multitude of operations that are involved in the program, but rather this flow chart is intended to provide an appreciation for the logical sequence of operations that are performed.

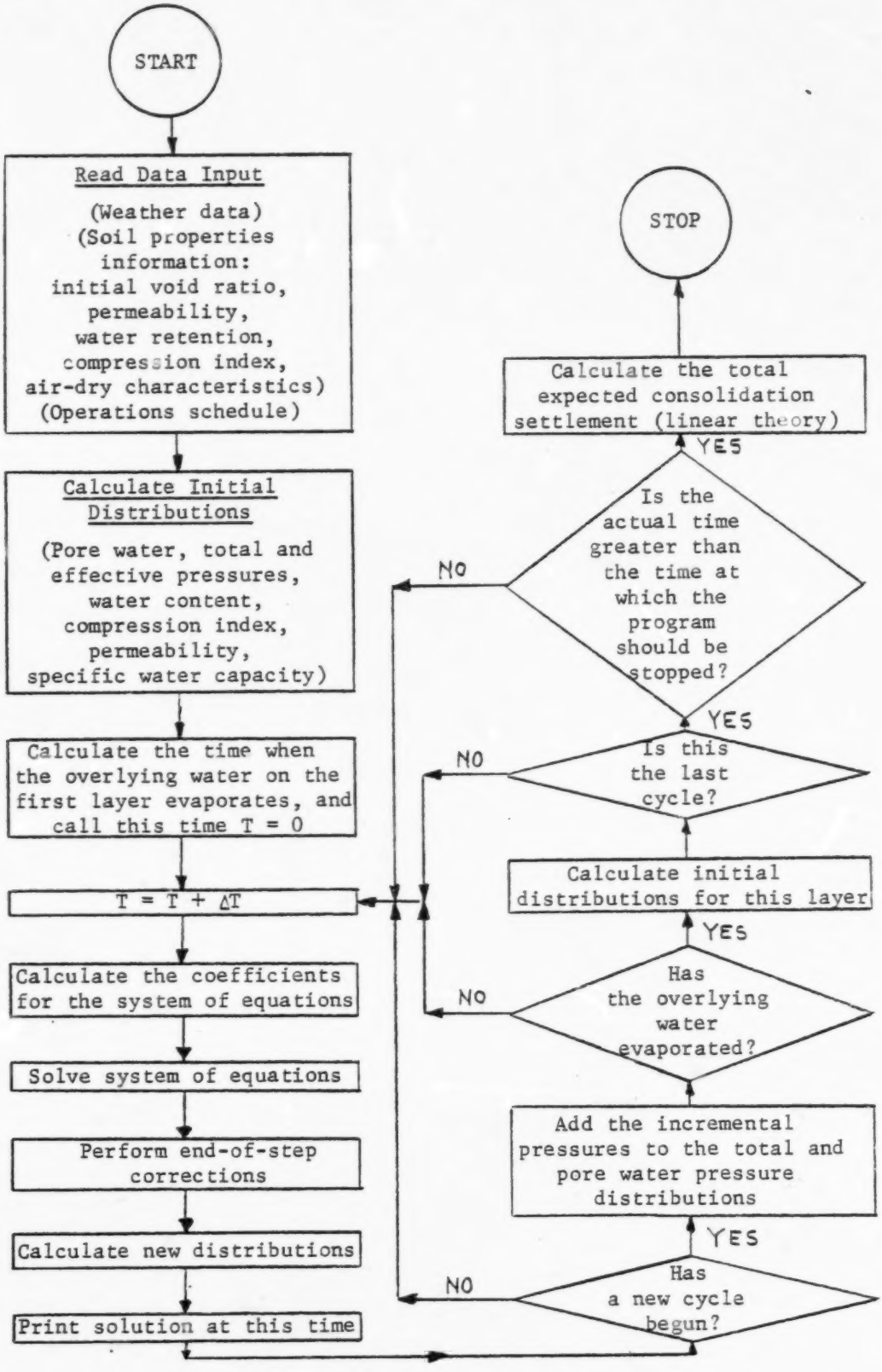
START
Read Data Input
(Weather data)
(Soil properties information:
initial void ratio,
permeability,
water retention,
compression index,
air-dry characteristics)
(Operations schedule)
Calculate Initial Distributions
(Pore water, total and effective pressures, water content, compression index, permeability, specific water capacity)
Calculate the time when the overlying water on the first layer evaporates, and call this time T = 0
T = T + ΔT
Calculate the coefficients for the system of equations
Solve system of equations
Perform end-of-step corrections
Calculate new distributions
Print solution at this time
STOP
Calculate the total expected consolidation settlement (linear theory)
YES
Is the actual time greater than the time at which the program should be stopped?
NO
YES
Is this the last cycle?
NO
Calculate initial distributions for this layer
YES
Has the overlying water evaporated?
NO
Add the incremental pressures to the total and pore water pressure distributions
YES
Has a new cycle begun?
NO

Figure A-1. Operational Flow Chart

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